A comment on Siegfried’s first lesson in econometrics

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A comment on Siegfried's first lesson in econometrics

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Abstract

In an insightful, and delightfully short, paper that was published in the Journal of Political Economy in 1970, Siegfried provided an important lesson about econometric methodology for all budding young econometricians. Unfortunately, there is a mistake in that paper. This short comment provides a simple correction for that mistake. This correction will hopefully enable econometrics students to obtain the fundamental insights from Siegfried's paper with less confusion than might otherwise have been the case.

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JEL Classifications: A20, C01, C10.

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Comment

In an insightful, and delightfully short, paper, Siegfried (1970) provided an important lesson for all budding young econometricians. This lesson covered an important principle of econometric methodology that is rarely, if ever, explicitly stated in econometrics textbooks. The principle in question might best be described as Occam's yeast. It makes the obvious, but often overlooked, point that when something is expressed as simply as possible, it is often too terse to be easily understood, interpreted or applied. The example provided by Siegfried is the very abstract statement that

$$1 + 1 = 2. \tag{1}$$

This statement is far too terse to be of much use to a practical person. As Siegfried notes, there are much less terse ways of writing this statement.

Unfortunately, in attempting to allow for the incorporation of dimensions larger than one into this statement, Siegfried made a mistake.[1] This mistake[2] appears in Equation (11) of Siegfried's paper, which is

$$\left[\left(X^T\right)^{-1} - \left(X^{-1}\right)^T\right]! = 1.$$

The problem with this equation is that $\left[\left(X^T\right)^{-1} - \left(X^{-1}\right)^T\right]$ is not a scalar. Siegfried obtains this equation in three steps. First, he points out that

$$0! = 1 \tag{2}$$

when the zero in question is a scalar. This claim is clearly true. Second, he points out that

$$\left[\left(X^T\right)^{-1} - \left(X^{-1}\right)^T\right] = 0. \tag{3}$$

While this claim is true, it needs to be noted that the zero in this equation is a square null matrix rather than a scalar. The third step in Siegfried's reasoning is the point at which the mistake is made. Siegfried substitutes

[1]If we ignore the mistake that is the focus of this comment, then there are also two typographical errors in Siegfried's Equation (12). These typographical errors involve the omission of the factorial sign and the exponent δ from the expression $\lim_{\delta\to\infty}\left(\left\{\left[\left(X^T\right)^{-1} - \left(X^{-1}\right)^T\right]! + \left(\frac{1}{\delta}\right)\right\}^{\delta}\right)$.

[2]Prof Siegfried has informed me that the mistake that is the focus of this paper was pointed out to him by a number of people shortly after the publication of Siegfried (1970). It should be noted that this mistake does not alter the fundamental insight that was conveyed by Siegfried (1970).

Equation (3) into Equation (2) to obtain his Equation (11). Unfortunately, this substitution is invalid because

$$\begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}_{(k\times k)} \neq (0)_{(1\times 1)}$$

when $k \neq 1$.

Fortunately, there is an easy modification that will correct for Siegfried's mistake and ensure the validity of the fundamental insights from Siegfried's paper. This correction involves taking the determinants of both sides of Equation (3) to obtain

$$\det\left(\left[\left(X^T\right)^{-1} - \left(X^{-1}\right)^T\right]\right) = 0. \tag{4}$$

Note that the zero in this equation is a scalar. As such, it is perfectly valid to substitute Equation (4) into Equation (2). Upon doing this, we obtain

$$\left\{\det\left(\left[\left(X^T\right)^{-1} - \left(X^{-1}\right)^T\right]\right)\right\}! = 1.$$

If this correction is implemented in Siegfried's paper from his Equation (11) onwards, then we obtain the following alternative representation of Equation (1):

$$\begin{aligned} &\ln\left(\lim_{\delta\to\infty}\left(\left[\left\{\det\left(\left[\left(X^T\right)^{-1} - \left(X^{-1}\right)^T\right]\right)\right\}! + \left(\frac{1}{\delta}\right)\right]^{\delta}\right)\right) \\ &+ \left(\sin^2(q) + \cos^2(q)\right) \\ = &\sum_{n=0}^{\infty} \frac{\cosh(p)\left(\sqrt{1-\tanh^2(p)}\right)}{2^n}. \end{aligned} \tag{5}$$

As Siegfried suggests, Equation (5) might be much easier to understand than Equation (1) because it is much less terse than Equation (1). Furthermore, it is possible that Equation (5) might be easier to apply than Equation (1) in an econometric setting because it explicitly incorporates matrices. We could, for example, imagine that

$$X = Z^T Z,$$

where Z is the design matrix from a linear regression model. Hopefully the correction proposed here will enable scholars to enjoy the marvellous insight into econometric methodology that is conveyed in Siegfried (1970) without the potential confusion that might have been caused by the unfortunate mistake that is contained in that paper.

References

[1] Siegfried, J. J. (1970), "A first lesson in econometrics", *The Journal of Political Economy 78(6)*, November-December, pp. 1378-1379.