Education Vouchers, Growth and Income Inequality*

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Abstract

This paper studies a growth model with public and private education alternatives. The impact of education vouchers for economic growth and the evolution of income inequality are considered. Results indicate that introducing education vouchers can increase economic growth. Households switching from public to private education experience higher incomes. This raises the tax base, in turn raising public education expenditures and growth of the whole economy. Vouchers are found to generally increase income inequality. Welfare comparisons show that voucher schemes may in some cases gain majority support, depending on assumptions and parameters. The results add a new dimension on which vouchers can be evaluated in the continuing policy debate.

JEL Classification: D31, H20, I22, J24, O40.

Keywords: Education Choice, Growth, Income Distribution, Vouchers.

1 Introduction

Reform of education and its financing have been topics of growing importance in policy debates over the last four decades in the US. Proposals for education vouchers have been increasing in number with numerous pilot programs in place. However, the debate over the beneficiaries of education vouchers continues. Typical issues include the benefits of school choice and competition and the problems of cream skimming the best students and greater inequality to name a few; Cohn (1997) provides a collection of works covering a wide range of issues relevant to education vouchers.

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This paper studies the implications of private education vouchers for economic growth and income inequality. Vouchers take the form of a uniform subsidy to all families using private schools. The policy issue is whether an existing public education budget be reallocated in a growth enhancing manner through the use of vouchers. Vouchers raise the incomes of private education households such that the tax base grows, in turn providing for increased public education and human capital accumulation for public education households; a fiscal externality.¹

The coexistence of public and private education leads to a bimodal income distribution. The effects of vouchers on the dynamic evolution of the economy are studied, though the endogenous distribution of income cannot be analytically characterized. Numerical simulations are used to investigate the implications of vouchers for growth and income inequality. The model is calibrated in accordance with the approach used to study vouchers in a static setting in Epple and Romano (1996), using data on US human wealth distribution estimated in Lillard (1977). An alternative calibration suggested by Cohen-Zada and Justman (2003) is also considered.

The simulation analysis identifies two main results. First, is that education vouchers increase per capita income and growth. Second, is that there is indeterminacy with respect to income inequality. Introducing vouchers increases inequality measured using Gini coefficients, though alternative measures identify the possibility of reduced inequality. Generalized Lorenz curves show that vouchers offer Pareto improvement in utilities, though among the early generations to experience vouchers, majorities may be worse off and prefer not to have vouchers in some cases. This is consistent with the US experience that vouchers struggle to gain support in referenda, indicating heavy discounting beyond current generations. The endogenous income distribution is found to exhibit bimodality, with the lower income group attending public schools and the upper income group attending private schools. This result has similarities to Galor and Zeira (1993). In this case, there is no credit market for human capital, while the additional assumption that leads to multiple equilibria or convergence clubs is that parents must choose between public and private education.²

The paper brings together two strands of the literature, first is the study of education
vouchers and second the investigation of links between growth, education and human capital accumulation. The literature on education vouchers is typically theoretical, due to the limited data available for empirical studies, and focuses on one period models, analyzing the static effects of vouchers on current students. This paper develops the voucher literature by considering the effects of vouchers on generations beyond the first to use them.

Voucher design and the efficient targeting of education funding are studied in Bearse et al. (2000) and Nechyba (2000). The key results are (i) vouchers can improve educational outcomes and (ii) appropriately targeting vouchers can provide further improvements in the distribution of educational expenditures and outcomes. Issues of targeting are not studied in this paper in order to (i) simplify the analysis and (ii) focus on the positive growth implications of the least beneficial form of vouchers, namely uniform private education vouchers.

Another important simplifying assumption is that the existing public education budget is available for reallocation between private education vouchers and public education for those that do not take up the private alternative while maintaining a balanced government budget. This eliminates voting issues from the model. While much of the existing literature investigates voting issues, in the overlapping generations framework outlined below, each generation’s voting problem would be a self contained, one period, optimization problem and the inclusion of social choice will not enrich the model’s dynamics or provide new insights into voting issues. Thus, the focus is on dynamic efficiency gains that can be extracted from the existing budget through the simple market mechanism of a uniform private education voucher.

Education vouchers are studied in a growth setting in Kaganovich and Zilcha (1999). Their work looks at the relationship between social security, education with vouchers and growth with the benefits of vouchers for growth depending on parental altruism for their child. While Kaganovich and Zilcha (1999) study a representative agent framework, this paper considers heterogeneous agents, thereby allowing for the study of income distribution and of dynamic interactions between public and private education students. Gradstein and Justman (1997) compare different forms of education finance, namely subsidizing private education and universal private education. They find that subsidizing private education
provides higher growth and inequality than universal public education, depending on an externality arising from the average level of human capital. Vouchers operate in a similar manner here. Public education is tax financed and depends on per capita income and in turn per capita human capital, thus the voucher increases a fiscal externality, the tax base, and enhances growth. An important difference between this paper and Gradstein and Justman (1997) is that they compare these different forms of finance whereas this paper incorporates them into an unified model, offering an explanation of income distribution that depends on the existence of both public and private education. Glomm and Ravikumar (1999) study a dynamic voucher economy where vouchers are available to all households and there is no public education. The voucher setup here provides subsidies to private education households only, facilitating the fiscal spillover which drives higher growth for both public and private education households. The lack of public education in their voucher model allows the introduction of social choice. However, tax rates are relatively stable, not changing much over time, supporting the present modelling assumption to abstract away from social choice.

Section 2 sets out the model to be studied. The model is solved and equilibrium characterized in Section 3. The effects of vouchers on growth and income inequality are studied in Sections 4 and 5 respectively. Numerical simulations are used in Section 6 to further analyze the implications of education vouchers on growth and income inequality. A summary of results is provided in Section 7.

2 A Model with Vouchers

Consider an economy of two period lived overlapping generations. The population is normalized to unity and is constant over time with each parent having only one child. Human capital endowments of the first generation follow some distribution $f_0(h)$. For subsequent generations, the human capital distribution is endogenous and given by $f_t(h)$ for the cohort that is old in period $t+1$. Parents (i) inelastically supply their unit time endowment; (ii) are taxed, with revenues used to finance education; and (iii) decide to send their child to a public or private school. If a public school is chosen, all after tax income is consumed and the child’s education is fully government provided. If private education is chosen, an
An education voucher is available and after tax income is allocated between private consumption and education.

Parents derive utility from the education provided to their child \(q_{i,t}\) and household consumption \(c_{i,t}\), where \(i\) and \(t\) indices identify respectively the agent’s type, based on human capital, and the period in which they are old. A Constant Elasticity of Substitution (CES) utility specification is assumed:

\[
U(c_{i,t}, q_{i,t}) = \left(\beta c_{i,t}^{-\rho} + (1 - \beta) q_{i,t}^{-\rho}\right)^{-\frac{1}{\rho}}
\]

where \(\beta \in (0, 1)\), \(\rho \in (-1, \infty)\). This preference specification is used in both Epple and Romano (1996) and Cohen-Zada and Justman (2003), and is chosen to facilitate calibration based on these previous studies.

A child’s human capital depends on the education expenditure made on their behalf and on their parent’s human capital. Human capital \(h_{i,t}\) evolves according to:

\[
h_{i,t} = \theta q_{i,t}^{\gamma} h_{i,t-1}^{1-\gamma}
\]

where \(\gamma \in (0, 1)\) and \(\theta > 0\). Individual human capital accumulation depends on education expenditures and levels of parental education.

Parents inelastically supply their unit time endowment at a wage given by their human capital with income \(y_{i,t}\) given by:

\[
y_{i,t} = h_{i,t-1}
\]

This assumption allows a clear focus on the implications of education expenditure and choices, exclusive of labour supply decisions, on income distribution.

Two schooling alternatives are available, public or private schooling. In the public education system, students receive identical, government funded education expenditures. The parent does not make any other contributions. If public education is chosen, the parent’s education expenditure and budget constraint are respectively given by:

\[
q_{i,t} = E_t
\]

\[
c_{i,t} = (1 - \tau) y_{i,t}
\]
where $\tau$ is the marginal tax rate imposed on all income. Since the model abstracts from voting issues, the tax rate is treated as exogenous and does not vary over time. Tax revenue is used solely to finance education, either public education or private education vouchers.

The proportion of the population in public education in period $t$ is given by $Q_t \in (0, 1)$ while per student public education expenditure is given by:

$$E_t = \frac{\tau Y_t - s Y_t (1 - Q_t)}{Q_t}$$

(6)

where $Y_t$ is both aggregate and average income and $(1 - Q_t)$ is the proportion of the population receiving private education. The voucher provided to parents choosing private education is given by:

$$S_t = s Y_t$$

(7)

where $s$ is the size of the voucher as fraction of average income, hereafter the subsidy rate. It is assumed that the subsidy rate is sufficiently small so that $E_t > S_t$. This link between vouchers and per capita income is a simplifying assumption. Various forms of private education subsidies are used around the world, in Australia, France and the Netherlands to name a few. Costs and the behavior of subsidies depend on the educational structure. In the Netherlands for example, average costs are increasing because of the proliferation in the number of smaller private schools; see Cohn (1997) and West (1997) for more details.

If the parent opts out of the public system, the child’s education is financed out of after tax income, which is supplemented by a private education voucher, $S_t$. This voucher can only be used at private schools and cannot be used to purchase consumption, implying that if private education is chosen, $q_{i,t} > S_t$. Private education expenditure and the parent’s budget constraint are given by:

$$q_{i,t} = e_{i,t} + S_t$$

(8)

$$e_{i,t} = (1 - \tau) h_{i,t} + S_t - q_{i,t}$$

(9)

where $e_{i,t}$ is the private education spending made by the parent; this is similar to the voucher framework used in Epple and Romano (1996).

The tax and the subsidy rates are determined by the government and individual agents treat them both as exogenous. The issue of interest here is whether a shift from an education
system where \( s = 0 \) to a system where \( s > 0 \), while maintaining \( \tau \) constant, can increase growth and reduce income inequality. The determination of either the tax or subsidy rates could be endogenized using majority voting, as in Epple and Romano (1996), Glomm and Ravikumar (1998) and Cardak (2001). Alternatively, both the tax and subsidy rates could be determined endogenously in a sequential voting framework as in Hoyt and Lee (1998). The lack of strategic interaction between cohorts or generations means that each generation’s voting problem would be a distinct static political optimization problem, the inclusion of which complicates the analysis and does not enrich the dynamics of the model.\(^6\)

3 Equilibrium Properties with Vouchers

A general equilibrium for the model described by equations (1) to (9) comprises a set of sequences of individual choices for consumption and education, \( \{c_{i,t}, q_{i,t}\}_t^{\infty} \), that maximize individual utility, given the tax and subsidy rates \((\tau, s)\). Equilibrium also requires that at the given tax and subsidy rates \((\tau, s)\) and per capita income \( \{Y_t\}_t^{\infty} \), the public education expenditure and enrolment, \( \{E_t, Q_t\}_t^{\infty} \), are such that the government budget balances, satisfying (6).

Optimal choices of a parent that sends their child to public school are:

\[
c_{i,t} = (1 - \tau) y_{i,t} = (1 - \tau) h_{i,t-1} \\
qu_{i,t} = E_t
\]

The dynamic evolution of human capital for a public education family is given by:

\[
h_{i,t} = \theta E_t^{\gamma} h_{i,t-1}^{1-\gamma} = \theta \left( \frac{Y_t (\tau - s + sQ_t)}{Q_t} \right)^{\gamma} h_{i,t-1}^{1-\gamma}
\]

which can be solved for a state dependant fixed point:

\[
h_{i}^{f,u} = \theta^{\frac{1}{\gamma}} \left( \frac{Y_t (\tau - s + sQ_t)}{Q_t} \right)
\]

This fixed point depends on aggregate state variables \( Y_t \) and \( Q_t \) and on the exogenous, government determined tax and subsidy rates, \( \tau \) and \( s \). The human capital of all students sent to public school in period \( t \) will converge to \( h_{i}^{f,u} \), though at different rates depending on their parents human capital, \( h_{i,t-1} \). This fixed point may change in the subsequent period.
as both $Y_{t+1}$ and $Q_{t+1}$ may differ from their period $t$ values, hence $h_{t+1}^{i,u}$ cannot strictly be referred to as a steady state.

The optimal choices of a parent choosing private education for its child are given by:

$$c_{i,t} = \frac{B \left( (1 - \tau) y_{i,t} + S_t \right)}{1 + B}$$

(14)

$$q_{i,t} = \frac{(1 - \tau) y_{i,t} + S_t}{1 + B}$$

(15)

where $B = \left( \frac{\beta}{1 - \rho} \right)^{\frac{1}{1 - \rho}}$. Private education can be an optimal choice for a parent only if:

$$(1 - \tau) y_{i,t} > BS_t$$

(16)

which ensures that $q_{i,t} > S_t$, a necessary condition for the optimality of the choice of private education, given that consumption cannot be purchased with vouchers and $E_t > S_t$.

Human capital evolves according to the following dynamic equation in the private education model:

$$h_{i,t} = \theta \left( \frac{(1 - \tau) h_{i,t-1} + sY_t}{1 + B} \right)^{\gamma} h_{i,t-1}$$

(17)

This dynamic equation provides some restrictions on parameters to be used in the simulation analysis. When $s = 0$, the dynamic equation is linear in $h_{i,t-1}$ and the coefficient, $\theta \left( \frac{1 - \tau}{1 + B} \right)^{\gamma}$, must be greater than unity to rule out trivial equilibria. When $s > 0$, equation (17) is strictly concave, however, it is not sufficiently concave for a fixed point to exist.\(^7\)

**Proposition 1** In equilibrium, assuming that $\theta \left( \frac{1 - \tau}{1 + B} \right)^{\gamma} > 1$ and $s > 0$, the dynamic equation for private education human capital satisfies the following conditions:

(i) \( \lim_{k \to 0} \frac{\partial h_{i,t}}{\partial h_{i,t-1}} \bigg|_{h_{i,t-1} = k} = \infty \) and

(ii) \( \lim_{k \to \infty} \frac{\partial h_{i,t}}{\partial h_{i,t-1}} \bigg|_{h_{i,t-1} = k} = \theta \left( \frac{1 - \tau}{1 + B} \right)^{\gamma} > 1 \)

The proof can be found in Cardak (2001). Thus no private education fixed point for human capital exists. The expansion path is not linear as in an AK growth model, but is instead concave while in the limit, as $h_{i,t-1} \to \infty$, it is weakly concave or approaches linearity. This behavior is illustrated in the top panel of Figure 1.
Each parent chooses the education system that provides greatest utility. This in turn depends on own income, the sizes of the tax and voucher rates, public school enrolment and per capita income. All of which determine the relative attractiveness of public and private education. To find an income threshold which separates the population into public and private education groups, substitute equations (10) and (11) into (1), and (14) and (15) into (1), equate and solve for the level of income where parents are indifferent between alternatives, \( y^*_t \). Though \( y^*_t \) cannot be explicitly characterized, the following result ensures its existence and uniqueness.

**Proposition 2** For a given tax and voucher rate, \( \tau \) and \( s \), public education expenditure, \( E_t \), and per capita income, \( Y_t \), there exists a unique income threshold \( y^*_t = y(E_t, Y_t; \tau, s) \) such that all parents with \( y_{i,t} \leq y^*_t \) prefer public education and those parents with \( y_{i,t} > y^*_t \) prefer private education.

The proof can be found in the appendix of this paper. The result means that the poor will prefer public education and the rich will prefer private education. It allows the proportion of the population in public education, \( Q_t \), to be uniquely determined for a given combination of \( (E_t, Y_t; \tau, s) \):

\[
Q_t(E_t, Y_t; \tau, s) = \int_0^{y^*_t} f_{t-1}(h) \, dh = F_{t-1}(y^*_t) \quad (18)
\]

where \( F_{t-1}(\cdot) \) is the cdf and \( f_{t-1}(\cdot) \) is the pdf of human capital accumulated in period \( t - 1 \). Alternatively, \( Q_t \) can be determined from equation (6).

## 4 Vouchers and Growth

The potential gains from the introduction of a private education voucher in this model can be illustrated with a simplified example. Consider an economy with three groups of homogeneous agents, the poor, the middle class and the rich, respectively comprising fractions \( Q_1 \), \( Q_2 \) and \( (1 - Q_1 - Q_2) \) of the population, with respective incomes \( y_{1,t} \), \( y_{2,t} \) and \( y_{3,t} \) such that \( y_{1,t} < y_{2,t} < y_{3,t} \). The poor and middle class attend public schools, while the rich attend private schools. In terms of \( y^*_t \) from Proposition 2, \( y_{1,t} \) and \( y_{2,t} \leq y^*_t = y(Y_t, Q_t = (Q_1 + Q_2); \tau, s = 0) < y_{3,t} \).
Consider the introduction of a private education voucher \( S_t = sY_t \) which causes only the middle class, \( Q_2 \), to switch to private education. The rich and middle class receive the voucher and attend private schools while the poor continue to attend public schools. It is now the case that \( y_{1,t} \leq y^*_{t} = y(Y_t, Q_t = Q_1; \tau, s) < y_{2,t} \) and \( y_{3,t} \). The voucher needs to be sufficiently large to make private education attractive to the middle class but not so large that the poor want to switch, while satisfying the feasibility constraint in equation (16), \((1 - \tau) y_{2,t} > sBY_t\).

Using equation (12) for public education and equation (17) for private education, the first generation of rich students to use the voucher are better off:

\[
h_{3,t}^V = \theta \left( \frac{(1 - \tau) h_{3,t-1} + sY_t}{1 + B} \right)^{\gamma} h_{3,t-1}^{1-\gamma} > \theta \left( \frac{(1 - \tau) h_{3,t-1}}{1 + B} \right)^{\gamma} h_{3,t-1}^{1-\gamma} = h_{3,t}^{NV} \tag{19}
\]

where the superscript \( V \) and \( NV \) denote cases with and without vouchers respectively. The value of \( s \) is chosen so middle class parents are better off. Middle class students will have higher human capital if:

\[
h_{2,t}^V = \theta \left( \frac{(1 - \tau) h_{2,t-1} + sY_t}{1 + B} \right)^{\gamma} h_{2,t-1}^{1-\gamma} > \theta \left( \frac{\tau Y_t}{Q_1 + Q_2} \right)^{\gamma} h_{2,t-1}^{1-\gamma} = h_{2,t}^{NV} \tag{20}
\]

Which places the following lower bound on \( s \):

\[
s > \frac{\tau (1 + B)}{(Q_1 + Q_2)} - \frac{(1 - \tau) h_{2,t-1}}{Y_t} \tag{21}
\]

For poor students, public education expenditures will benefit from the reduction in public school enrolments from \((Q_1 + Q_2)\) to \(Q_1\), but the cost is the public education budget will be reduced because of voucher expenditure. Poor students will have higher human capital under the voucher scheme if:

\[
h_{1,t}^V = \theta \left( \frac{\tau Y_t - sY_t (1 - Q_1)}{Q_1} \right)^{\gamma} h_{1,t-1}^{1-\gamma} > \theta \left( \frac{\tau Y_t}{Q_1 + Q_2} \right)^{\gamma} h_{1,t-1}^{1-\gamma} = h_{1,t}^{NV} \tag{22}
\]

which only occurs if:

\[
sY_t (1 - Q_1) < \frac{\tau Y_t}{(Q_1 + Q_2)Q_2} \tag{23}
\]

This condition requires the voucher scheme to raise per student public education expenditures, \( E_t \), and occurs when the costs of the voucher program (LHS) are lower than the
benefits of voucher (RHS) in terms of the public education budget. If either the size of the voucher or the size of the rich and middle classes or both are sufficiently small, then all students, rich, middle class and poor will have higher human capital with the introduction of the voucher. Relaxing the homogeneity assumption, similar conclusions arise. If the current private education user group is small and the numbers of people switching is also small, the introduction of a private education voucher can provide all households with higher human capital.

If all households have higher human capital after the introduction of vouchers, per capita income in the next period must be higher, which will provide further growth to subsequent generations of poor students, through a tax base driven improvement in public education, a fiscal spillover. It is not clear whether all households benefit in the long run if the human capital of poor students is initially lowered by vouchers, this is investigated using simulations below.

5 Vouchers and Income Inequality
Endogenous income distribution cannot be analytically characterized in this model due to the discrete education choices. Parts of the income distribution are characterized below using within group comparisons, this is later supplemented with simulations.

Focusing on households that attend public schools whether private education vouchers are available or not, the relevant dynamic equation is (12). Comparing two families, \( j \) and \( k \), in this group, the ratio of their incomes is:

\[
\frac{h_{j,t}}{h_{k,t}} = \theta \left( \frac{Y_{t}(\tau - s + sQ_{t})}{Q_{t}} \right) ^{\gamma} \frac{h_{j,t-1}^{1-\gamma}}{h_{k,t-1}^{1-\gamma}} = \left( \frac{h_{j,t-1}}{h_{k,t-1}} \right) ^{1-\gamma}
\]

Incomes within this group converge over time, as \( \gamma \in (0, 1) \). Introducing private education vouchers will not change the rate at which incomes converge among those who continue to use public education, as vouchers change public education identically for all public education students. This changes income levels by a constant scale factor, leaving relative inequality within this group unchanged.

The relevant dynamic equation for households that use private education both before and after the introduction of vouchers is (17). Comparing families \( l \) and \( m \) from this group,
the income ratio is:

$$\frac{h_{l,t}}{h_{m,t}} = \frac{\theta \left( \frac{(1-\tau)h_{l,t-1}+sY_t}{1+B} \right)^\gamma}{\theta \left( \frac{(1-\tau)h_{m,t-1}+sY_t}{1+B} \right)^\gamma} \frac{h_{l,t-1}^{1-\gamma}}{h_{m,t-1}^{1-\gamma}} = \left( \frac{(1-\tau)h_{l,t-1}+sY_t}{(1-\tau)h_{m,t-1}+sY_t} \right)^\gamma \left( \frac{h_{l,t-1}}{h_{m,t-1}} \right)^{1-\gamma}$$ (25)

In the case without vouchers, $s = 0$, relative inequality within this group does not change over time.

Introducing private education vouchers leads incomes within this group to converge over time. Assuming $h_{l,t-1} > h_{m,t-1}$, changing the subsidy rate from $s = 0$ to $s > 0$ reduces the income of household $l$ relative to household $m$ from period $t - 1$ to period $t$:

$$\frac{h_{l,t}}{h_{m,t}} < \frac{h_{l,t-1}}{h_{m,t-1}}$$ (26)

This result arises because vouchers reduce the relative differences in education expenditures within this group of private education users.

The effects of vouchers on mobile groups (switching from public to private education) has not been considered because the endogenous income distribution cannot be fully characterized. Simulations are used to further characterize the impact of vouchers below.

6 Dynamic Analysis

6.1 Calibration

The model is calibrated to match aggregate variables and behavior observed in the US, following the methodology of Epple and Romano (1996). Simulations are used to illustrate the endogenous distribution of income for cases with and without vouchers. The approach is to simulate the model for a large sample and use the income distribution of the sample as an approximation to the actual endogenous income distribution. Economic variables are plotted over time to illustrate the differences between the cases with and without vouchers.

Since the model is of overlapping generations and $y_{i,t} = h_{i,t}$, the first generation’s income distribution is calibrated to human wealth distribution in the US, as suggested in Glomm and Ravikumar (1999). The figures used are based on Lillard’s (1977) estimation of human wealth distributions in 1970, where mean and median human wealth (measured in thousands) are respectively $166.99$ and $151.39$. These figures are adjusted to 1996 prices using the Consumer Price Index (CPI), with a mean of $515.04$ and median of $466.93$. Assuming
lognormality implies that \( \ln (h_{i,0}) \sim N(\mu, \sigma^2) \), with \( \mu = 6.146 \) and \( \sigma = 0.443 \), where \( h_{i,0} \) are the human capital endowments of the first generation.

Public education expenditure in 1996-97 was $5882 per student. To be consistent with the human wealth calibration above, the cost of 12 years of education is treated as education expenditure. Thus per household public education expenditures are set to $35292 for the first generation. This assumes approximately 0.5 students per household; see Epple and Romano (1996) and Bearse et al (2000). The tax rate is chosen to finance the education budget and maintained at this rate for the full time horizon of the simulations. The model parameters, \( \beta \) and \( \rho \) are chosen so the model endogenously allocates 12% of enrolments to private education in the first period, as observed in the US in 1996-97. These data are drawn from Public Elementary-Secondary Education Finance Data: 1996-97, US Census Bureau (2000) and the Digest of Education Statistics 1999, National Center for Education Statistics (2000).

Two combinations of preference parameters \((\rho, \beta)\) are used to investigate the model. The first (Case A) is the benchmark case used in Epple and Romano (1996), where \( \rho = 0.54 \) and \( \beta = 0.967 \), which leads to private education enrolments of 12% and implies an elasticity of substitution of \(-0.65\). The second case (Case B) to be studied is the case where \( \rho = -0.359 \) and \( \beta = 0.750 \), also providing private education enrolments of 12% with an elasticity of substitution of \(-1.56\). This parameterization is suggested in Cohen-Zada and Justman (2003) who present a unified theory-estimation-calibration-simulation approach to a similar one period model.\(^{12}\)

In the calibration of the human capital production function, \( \theta \) is chosen so that \( \theta (\frac{1-1}{1+B})^\gamma > 1 \), as discussed in Section 3, and that aggregate growth is around 2% per year. Assuming a generation comprises 30 years, this amounts to 81% growth per period. While a precise value for \( \gamma \) is difficult to pin down and open to debate, empirical studies suggest that \( \gamma < 0.5 \). Glomm and Ravikumar (1998) simulate a model with a similar human capital production function and assume \( \gamma \in (0, 0.15) \). The results reported here will focus on the case where \( \gamma = 0.1 \), while sensitivity analysis of the results will include the cases where \( \gamma = 0.01 \) and 0.2. The sample size used to approximate the distribution of income is chosen to be 10000 while
the model is simulated for 50 generations. Details of the simulation process are contained in Cardak (2001).

6.2 Results
Three vouchers are considered, with $S_0 = 0, 12000, 24000$ (for 12 years of schooling). These absolute voucher values are allowed to grow over time, while maintained as a constant proportion of average human wealth. In Figures 2 and 4 below, Case A is considered, $\rho = 0.54$, $\beta = 0.967$, $\theta = 2.33$, $\gamma = 0.1$, while in Figures 3 and 5, Case B is considered, $\rho = -0.359$, $\beta = 0.750$, $\theta = 2.304$, $\gamma = 0.1$.

6.2.1 Case A

Human Wealth and Growth: Vouchers raise per capita human wealth for Case A, see panel (a) of Figure 2, which shows ratios of per capita human wealth with vouchers to per capita human wealth without vouchers for vouchers of $S_0 = 12000$ (solid curve) and $S_0 = 24000$ (dashed curve). Private education voucher will immediately raise average human wealth. After 5 generations, average human wealth can be 3.2% higher if $S_0 = 24000$.

The ratios of the poorest and richest family’s human wealth with vouchers to human wealth without vouchers for $S_0 = 12000$ (solid curve) and $S_0 = 24000$ (dashed curve) are presented in panels (b) and (c) of Figure 2 respectively. The poorest family always has higher human wealth with vouchers. The increase in human wealth is initially smaller with the larger voucher. This is due to initial reductions in public education expenditure. Panel (c) of Figure 2 indicates that the wealthiest family always has higher human wealth with vouchers than without. This identifies the poor as being among the beneficiaries from private education vouchers. These gains arise from endogenous increases in public education expenditure.

Annual growth rates for per capita human wealth are presented in panel (d) of Figure 2 for $S_0 = 0$ (solid curve), $S_0 = 12000$ (dashed curve) and $S_0 = 24000$ (short dashed curve). Growth rates are up to 0.9% higher with vouchers (when $S_0 = 24000$). This results from the larger number of private education families, providing a greater tax base and enhancing the fiscal spillover that drives the endogenous growth of the public education sector.
Income Distribution, Inequality and Welfare: Panel (e) of Figure 2 shows ratios of Gini coefficients with vouchers to Gini coefficients without vouchers for $S_0 = $12000 (solid curve) and $S_0 = $24000 (dashed curve). Vouchers increase human wealth inequality measured by Gini coefficients. After 5 generations, Gini coefficients are up to 5.5% higher with vouchers than without. Inequality increases because families switching from public to private education are moving further into the upper tail of the human wealth distribution, moving further above the mean and raising overall human wealth inequality.

Human wealth inequality is also measured by the ratios of voucher to no voucher cases of the ninth to the first decile human wealth ratio and is plotted in panel (f) of Figure 2 for $S_0 = $12000 (solid curve) and $S_0 = $24000 (dashed curve). The ninth to first decile ratios suggest vouchers lead to very little change in human wealth inequality. This implies that while vouchers may increase Gini coefficients, panel (e), the extrema of the distribution are largely unaffected. The main changes in the distribution are between the 82nd and 88th percentiles, those switching education regimes.

Further exploring inequality, Lorenz and generalized Lorenz curves (GLC’s) for the distribution of human wealth after 5 generations are presented in panel’s (a) and (b) of Figure 4, with $S_0 = $0 represented by the solid curve and $S_0 = $24000 represented by the dashed curve. The Lorenz curve in panel (a) is divided into 3 regions, (i) below the vertical line at 0.82 are those households that attend public schools with and without vouchers, (ii) between 0.82 and 0.88 are the households that use public schools without a voucher and private schools with a voucher, the mobile families and (iii) above the vertical line at 0.88 are those households who always attend private schools. This Lorenz curve confirms the analytical result that relative inequality within the public education group does not change with the introduction of vouchers, equation (24). This can be seen by the linear portion of the Lorenz curve, which measures inequality within the public education group. Panel (a) also shows changes in shares of human wealth. Those that stay in public schools earn a smaller share of total wealth while those that were in private schools previously also end up with a smaller share of total wealth. The mobile group who switch from public to private education, have a greater share of total wealth.
The GLC’s in panel (b) have welfare implications. As the GLC for the case with \( S_0 = $24000 \) lies above the GLC for the case with \( S_0 = $0 \), the human wealth distribution with vouchers stochastically dominates the distribution without vouchers. The case where \( S_0 = $12000 \) is not presented but the GLC lies entirely between the GLC’s for the no voucher and \( S_0 = $24000 \) cases. This implies all agents are better off with vouchers in terms of utility, thus, after 5 generations, vouchers offer a welfare improvement.

This raises the question of how long it takes for vouchers to gain majority support. Individual utilities under the two alternative voucher values are calculated and compared to the case with no vouchers. Comparisons indicate that a voucher policies with \( S_0 = $12000 \) gains immediate majority support while a voucher policy with \( S_0 = $24000 \) gains majority support by the third generation. The results for the larger voucher are consistent with the fact that in the US, referenda proposing the introduction of vouchers repeatedly fail to attract majority support.

Human wealth distribution in the mixed education model, both with and without vouchers, is bimodal. Vouchers lead to a larger private education sector (upper income group) and a smaller gap between the human wealth of the public and private education groups. These distributions are not presented but examples can be found in Cardak (2001).

**Education Spending and Enrollment:** Ratios of total per student education expenditures with vouchers to per student education expenditures without vouchers are presented in panel (g) of Figure 2 for the cases of \( S_0 = $12000 \) (solid curve) and \( S_0 = $24000 \) (dashed curve) while the same ratios for per student public education expenditures are presented in panel (h) of Figure 2. Private education vouchers immediately increase total education expenditures. After 5 generations the total education budget is 4% higher in the case where \( S_0 = $12000 \) and 8.5% higher in the case where \( S_0 = $24000 \). Panel (h) shows that per student public education expenditures initially suffer from the introduction of vouchers. In the case where \( S_0 = $24000 \), they are 2% lower than in the no voucher case, but after 3 generations, the increase in the tax base has raised per student public education expenditures and after 5 generations, public education expenditures are 4% higher than they would have
been without a voucher.

The higher total education expenditure in panel (g) is the key source of the increased growth after the introduction of private education vouchers. Higher education spending results from two factors, first is the initial increase in total private education spending due to the voucher shifting more people into private education. Although average private education expenditure decreases, the larger numbers in private education raises total education expenditure.\(^\text{15}\) The second effect is that the incomes of agents who have switched from public to private education are now rising, rather than moving down towards the public education fixed point, (13). This increases per capita human wealth, verified by panel (a), thereby raising the tax base and per student public education expenditure, as seen in panel (h), further contributing to growth in per capita income.

In panel (i) of Figure 2, the proportion of the population in public education is presented for the case of no vouchers or \(S_0 = 0\) (solid curve), \(S_0 = 12000\) (dashed curve) and \(S_0 = 24000\) (short dashed curve). Larger vouchers entice larger numbers of people out of the public education system, with the lowest public education enrolments in the case of \(S_0 = 24000\). The \(S_0 = 24000\) voucher reduces public school enrolment from 88% in the case without a voucher to 81.8%. These changes in enrolment are larger than those found in Epple and Romano (1996) because taxes are held fixed at the rate required to balance the public education budget in the case without vouchers. Once vouchers are introduced, the government budget is still required to balance but taxes do not rise, making private education more attractive (or relatively cheaper) and drawing more students into private education than observed in Epple and Romano (1996).\(^\text{16}\)

6.2.2 Case B

Case B assumes \(\rho = -0.359, \beta = 0.750, \theta = 2.304, \gamma = 0.1\). For Case B, intergenerational altruism is higher because the elasticity of substitution between education and consumption is higher and the weight on the consumption good is smaller. Hence, education is more important to parents who are more willing to substitute education for consumption, relative to Case A. Analysis of Case B is presented in Figures 3 and 5, and closely follows that of Case A.
Per-Capita Income and Growth: Panel (a) of Figure 3 shows the effects of vouchers on per capita human wealth for Case B, illustrating that vouchers immediately offer increased per capita human wealth in Case B. The impact on per capita human wealth is larger than in Case A and after 5 generations, per capita income is 3.5% higher when $S_0 = $12000 and 7% higher when $S_0 = $24000.

Panel (b) of Figure 3 shows that for Case B, the poorest family’s human wealth is higher with vouchers than without vouchers. After 5 generations, the poorest family’s human wealth is increased by 0.78% when $S_0 = $12000 and by 1.5% when $S_0 = $24000. Panel (c) of Figure 3 shows the richest family’s human wealth is always higher with vouchers, by 0.08% when $S_0 = $12000 and by 0.16% when $S_0 = $24000. The poor benefit more from vouchers in Case B because the elasticity of public enrollments with respect to vouchers is higher than in Case A, this is discussed further below.

Growth rates for Case B are presented in panel (d) of Figure 3 and show the initial increase in growth to be greater than in Case A. When $S_0 = $24000 the initial growth rate is 2.40% higher than for the case without vouchers, while for $S_0 = $12000 the initial growth rate is 1.2% higher. After 5 generations, these differentials are 2.2% and 1.1% respectively.

Income Distribution, Inequality and Welfare: The ratios of Gini coefficients are presented in panel (e) of Figure 3. As in Case A, vouchers raise the Gini coefficient, initially by 1.2% for $S_0 = $12000 and 2.3% for $S_0 = $24000. After 5 generations, Gini coefficients are 4.3% and 8.2% higher for the respective vouchers.\(^{17}\)

The ratios of voucher to no voucher cases of the ninth to the first decile income ratio are plotted in panel (f) of Figure 3. These ratios show, as in Case A, very little change due to vouchers. In contrast to Case A, these ratios fall with the introduction of vouchers and the larger voucher $S_0 = $24000 reduces inequality by more than the smaller voucher, implying that for Case B, the extrema of the distribution are brought closer together by vouchers.

The Lorenz and GLC’s for the distribution of income after 5 generations for Case B are presented in panel’s (a) and (b) of Figure 5. The Lorenz curves show that the voucher increases the share of total income of families that switch from public to private education, primarily at the cost of the wealthy, those that were in private education without vouchers,
while those that remain in public education suffer only small reductions in their share of total income. The fact that the Lorenz curves with and without vouchers cross implies alternative measures of inequality that contradict the Gini coefficient can be found, consistent with results in panel (f) of Figure 2; see Atkinson (1970) for a detailed discussion.

The GLC’s in panel (b) imply the distribution of human wealth when $S_0 = $24000 stochastically dominates the human wealth distribution when $S_0 = $0. Thus, after 5 generations for Case B, all agents are better off with vouchers, a unanimous welfare improvement.

Comparing individual utilities under the no voucher case with the two alternative voucher values, it is again found that vouchers obtain immediate majority (unanimous) support for both $S_0 = $12000 and $S_0 = $24000. Typically, political support for voucher schemes has been weak, contradicting these results. One possible explanation is that in the model, all households have school age children, thereby ignoring issues of the actual demographic profile of the voting population.

Human wealth distributions for Case B are bimodal, as in Case A. The extrema of these distributions are closer together, panel (f) of Figure 3, implying lower inequality with larger vouchers. Examples of these distributions can be found in Cardak (2001).

**Education Spending and Enrollment:** The ratios of total per student education expenditures with vouchers to per student education expenditures without vouchers are presented in panel (g) of Figure 3 while the same ratios for per student public education expenditures are presented in panel (h) of Figure 3. The differences in education expenditures for Cases A and B explain the differences in growth and income inequality in the two cases.

Panel (g) of Figure 3 shows that vouchers immediately raise total education expenditures as in Case A, but by greater amounts, by 6.4% where $S_0 = $12000 and 14% for $S_0 = $24000. After 5 generations, education expenditures are up to 9.4% higher for $S_0 = $12000 and 20.5% higher for $S_0 = $24000.

Panel (h) shows that for Case B, both vouchers raise per student public education expenditures for every generation. Initially the rise is 1.2% $S_0 = $12000 and 2.0% for $S_0 = $2000, while after 5 generations the increases are 4.0% and 8.0% respectively. The public education
system experiences a greater benefit from the introduction of vouchers in Case B than in Case A.

Public education enrollments are illustrated in panel (i) of Figure 3. Enrollments are more responsive to vouchers in Case B; more households switch from public to private education for a given voucher. Public enrollments fall by 4.1% for $S_0 = $12000, by 9.8% for $S_0 = $24000, the falls are 3.0% and 7.1% respectively for Case A. The larger this elasticity of public enrollments with respect to vouchers is, the smaller the reduction in public education expenditures per student because the more students that switch, the fewer public education students the residual public education budget has to be shared amongst. Thus in Case B, public education expenditure is higher with the introduction of vouchers, leading to higher incomes, explaining the results in panels (a) through (d) of Figure 3.

6.3 Sensitivity Analysis

6.3.1 Production Parameters

Cases A and B are analyzed using the alternative values of $\gamma = 0.01$ and $\gamma = 0.2$. The results are qualitatively similar to those for Cases A and B considered above, with $\gamma = 0.1$. The main difference is that the increased growth from the introduction of vouchers is lower in the case where $\gamma = 0.01$ and higher in the case where $\gamma = 0.2$. This is to be expected, as a smaller (larger) value for $\gamma$ reduces (increases) the importance of education expenditure in human capital production. Vouchers will generate smaller spillovers and be less beneficial for growth if education spending is less important in the accumulation of human capital. This shows up in both Cases A and B in higher (lower) human wealth and education expenditure when $\gamma = 0.2$ ($\gamma = 0.01$). Inequality is largely unchanged except that the 9th to 10th decile ratios in both cases with $\gamma = 0.2$ show reduced inequality, again consistent with the increased growth arising from vouchers. Welfare comparisons indicate Case B is not sensitive to changes in $\gamma$ and vouchers continue to receive unanimous support. For Case A increasing $\gamma$ to 0.2 provides greater support for vouchers while reducing $\gamma$ to 0.01 reduces support for vouchers with $S_0 = $24,000 not supported by a majority until the eighth generation.

6.3.2 Preference Parameters
Two additional combinations of preference parameters \((\rho, \beta)\) are considered, with \(\rho = 1.00\) and \(\beta = 0.991\) \((\theta = 2.34, \gamma = 0.1)\) and substitution elasticity of \(-0.50\), and \(\rho = -0.70\) and \(\beta = 0.59\) \((\theta = 2.275, \gamma = 0.1)\) with a substitution elasticity of \(-3.33\). In both cases, private education enrolments are 12%. The results are again qualitatively similar to those for Cases A and B, however, the size of the impact of vouchers on growth does depend on the elasticity of substitution. When \(\rho = -0.70\) (1.00), the impact of vouchers on growth is stronger (weaker), relative to Cases A and B. This shows up in higher public and total education expenditures when \(\rho = -0.70\). This difference arises from the increased substitutability between education and consumption with \(\rho = -0.70\), leading to higher education expenditures, human capital growth and and spillovers arising from vouchers. In Cases A and B, vouchers increase Gini coefficients, this is unchanged for the alternatives considered here: Gini coefficients are higher (lower) than in Cases A and B when \(\rho = -0.70\) (1.00). Conversely, vouchers reduce (increase) 9th to 1st decile human wealth ratios relative to Cases A and B when \(\rho = -0.70\) (1.00). Welfare comparisons for the case where \(\rho = -0.70\) and \(\beta = 0.59\) indicate unanimous support for vouchers, as in Case B. For the case where \(\rho = 1.00\) and \(\beta = 0.991\), both the large and small vouchers receive majority support only from the fourth generation onwards.

7 Conclusions

Increased economic growth has been largely ignored as a potential benefit of education vouchers. In a setting where households can opt out of public education in preference for private education, private education vouchers have been shown to offer increased economic growth. Taxes were held constant and it was shown that a given public education budget can be redistributed through the use of private education vouchers in a way that will increase per-capita human wealth and in some cases increase the human wealth of all households.

Private education vouchers generated increased economic growth through a fiscal spillover. The tax base grew through a redistribution of the wealthier public education students into the private education system where they accumulated greater amounts of human capital. This drove increases in public education expenditure, generating growth for the students remaining in public education. Similar growth enhancement might be generated by ability
tracking or selective entry schools, however, such systems require some decision rule on how to select students. Vouchers offer an endogenous market mechanism through which these results might be generated. Enriching the dynamic model studied here with heterogeneous abilities and recursive preferences might shed light on whether a voucher system would be more efficient at enhancing growth than a system of ability tracking. This is reserved for future research.

While it was found in some cases the introduction of vouchers unanimously increased utility, this was not always the case. The first generation might be averse to private education vouchers as they may reduce current public education spending, only delivering public education improvements to subsequent generations. One possible way by which this aversion might be diminished is to means test voucher entitlements. This was found to generate greater political support for vouchers in a static setting in Bearse et al. (2000).

The model abstracted from the issues of between school competition and peer effects, important factors in the voucher debate. Incorporating competition for students with vouchers is argued to generate better educational outcomes for students in both private and public schools. This would increase growth in both the public and private education systems, strengthening the increased growth identified here.

Extending the model to incorporate peer effects may be perceived to weaken or overturn the results. This is not necessarily the case. While the best or most able peers are most likely to leave public schools due to the introduction of a private education voucher, as in Epple and Romano (1998), growth prospects would improve in the private education system and be harmed in the public system. In a peer effect-growth setting, the operative mechanism in this paper, the fiscal spillover that improves public education, would be strengthened and the voucher should provide better public education and enhance the growth of the whole economy. However, the public education group may suffer larger losses in the early stages of a voucher program, as they lose their best peers along with public education funds. In addition, inequality is likely to increase more than identified here, at least in the short run. This is an obvious direction for further research on the implications of vouchers for growth.
References


Endnotes

1 Nechyba (1999) identifies a similar fiscal spillover in a multidistrict model with public and private schools and migration between districts.

2 For a further discussion of dynamic models with multiple equilibria, club convergence and income distribution, see Galor (1996).

3 One of the few empirical studies of the effects of vouchers is Rouse (1998).

4 Other examples of growth models where public and private education alternatives are studied as distinct models include Eckstein and Zilcha (1994), Glomm and Ravikumar (1992) and Gradstein and Justman (2000).

5 The corner solution \( q_{i,t} = S_t \) cannot be an equilibrium because of the assumption that \( E_t > S_t \), so that a household considering the corner solution would always prefer public education with \( q_{i,t} = E_t \).
Developing the model in a way such that incorporating voting leads to strategic interaction between cohorts is the subject of ongoing research but is beyond the scope of the present study.

Deriving a fixed point for equation (17) yields \( h^{f,r}_t = \frac{\theta^{\frac{1}{1+B-\theta^+(1-\tau)}}}{1+B-\theta^+(1-\tau)} Y_t. \) If we impose the assumption that \( \theta \left( \frac{1-\tau}{1+B} \right)^\gamma > 1, \) then \( h^{f,r}_t < 0, \) which makes no economic sense and is not considered further.

This assumption has implications for the value of \( s. \) Recall that the value of \( y^*_t \) comes from an indifference between the use of the public or private education alternatives, see Proposition 2 and its proof. Since, in this case, \( y_{1,t}, y_{2,t} \) and \( y_{3,t} \) are given, \( s \) must be chosen so that the assumption holds. While \( y_{1,t} \leq y^*_t \) and \( y^*_t < y_{2,t} \) respectively place upper and lower bounds on \( s, \) these bounds can only be characterised implicitly due to the utility function assumed in equation (1).

In addition to the conditions outlined in footnote 8, equations (16) and (23) identify an upper bound for \( s \) while equation (21) provides a lower bound. Together these conditions define, for this example, the feasible space for \( s. \) This space cannot be analytically simplified to explicit lower and upper bounds, however the simulations employed below confirm the non-emptiness of this space.

If the voucher is too small, however, the middle class may not be induced out of the public education system and the gains will not be achieved.

Benabou (1996) uses a richer model with local and global linkages. While the voucher reduces within group inequality, it has the potential to widen the gap between groups. In a model like Benabou’s, with economy wide production complementarities, the resource reallocation of a voucher could lead to lower growth outcomes, depending on the relative importance of local and global interactions and the change in the distribution of human capital.

They assume private education is more efficient at providing education services, which leads to a higher value for \( \beta = 0.846 \) in order to match US private enrollment data. Given that it is assumed here that public and private education are equally efficient, a value of \( \beta = 0.750 \) results in enrollments matching US data and is used in the simulations below where \( \rho = -0.359. \)

Linearity of a portion of a Lorenz curve implies perfect equality within that subgroup.

See Cardak (2004) for an example of this without vouchers.

In the case with vouchers, the same government budget is expended on education while the families that
switch to private education spend more on education as do those that were already in private education.

16 The key reason for this difference is that Epple and Romano (1996b) endogenise the tax rate, as their primary interest is in their theoretical voting equilibrium results established earlier in their paper.

17 In the very long run, after 50 generations, the inequality results do appear a little different, with vouchers able to reduce inequality. These results are not presented here because of space constraints and the fact that 50 generations equates to 1500 years, a time horizon of relatively limited policy interest; details can be found in Cardak (2001) or on request from the author.
Equation (17) with voucher, slope asymptotes to $\hat{\theta}(1 - \tau)/(1 + B)^{\gamma}$.

Equation (17) with no voucher, slope of $\hat{\theta}(1 - \tau)/(1 + B)^{\gamma}$.

Equation (12) with voucher, after spillover raises fixed point.

Equation (12) with no voucher.

Equation (12) with voucher.

Figure 1: Illustration of dynamics of human capital given by equations (12), (13) and (17). Top panel shows private education case and bottom panel shows public education case. Bottom panel also shows the effects of the voucher generated fiscal spillover on public education (after $k$ periods), raising the growth path.
Figure 2: Time series of per-capita, poorest family's and richest family's human wealth, (a), (b) and (c) respectively. Growth rates for per capita human wealth, (d). Human wealth inequality, measured by Gini coefficient, (e), and by ratios of ninth to first decile income, (f). Education expenditure, including all vouchers, (g), per student public education expenditure, (h), and public education enrollments, (i). Ratios of the voucher cases to the no voucher case in panels (a) to (c) and (e) to (h), with $S_0 = $120000 (solid curve) and, $S_0 = $240000 (dashed curve). In panels (d) and (i), plots are for the cases $S_0 = $0 (solid curve), $S_0 = $120000 (dashed curve) and $S_0 = $240000 (short dashed curve). Parameter values for Case A, $\theta = 2.33$, $\gamma = 0.1$, $\rho = 0.54$, $\beta = 0.967$. 
Figure 3: Time series of per-capita, poorest family's and richest family's human wealth, (a), (b) and (c) respectively. Growth rates for per capita human wealth, (d). Human wealth inequality, measured by Gini coefficient, (e), and by ratios of ninth to first decile income, (f). Education expenditure, including all vouchers, (g), per student public education expenditure, (h), and public education enrollments, (i). Ratios of the voucher cases to the no voucher case in panels (a) to (c) and (e) to (h), with $S_0 = $12000 (solid curve) and, $S_0 = $24000 (dashed curve). In panels (d) and (i), plots are for the cases $S_0 = $0 (solid curve), $S_0 = $12000 (dashed curve) and $S_0 = $24000 (short dashed curve). Parameter values for case B, $\theta = 2.304$, $\gamma = 0.1$, $\rho = -0.359$, $\beta = 0.750$. 
Figure 4: Lorenz curves, panel (a), and generalised Lorenz curves, panel (b), for income distributions with no vouchers, $S_0 = 0$ (solid curves) and with vouchers, $S_0 = 24000$ (dashed curves), after 5 generations. Parameter values for Case A, $\theta = 2.33, \gamma = 0.1, \rho = 0.54, \beta = 0.967$.

Figure 5: Lorenz curves, panel (a), and generalised Lorenz curves, panel (b), for income distributions with no vouchers, $S_0 = 0$ (solid curves) and with vouchers, $S_0 = 24000$ (dashed curves), after 5 generations. Parameter values for Case B, $\theta = 2.304, \gamma = 0.1, \rho = -0.359, \beta = 0.750$. 

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Appendix

Proof of Proposition 2

Proposition 2 is proved in three steps. The utility functions have been subjected to a monotonic transformation, raised to the power of \(-\rho\) in order to simplify the analysis and presentation. First, it is shown that there exists a level of income such that utility under public education exceeds utility under private education. Consider the case \((1 - \tau) y_{i,t} = BS_t\), utility under public education is given by:

\[
U_u(y_{i,t} = \frac{BS_t}{(1 - \tau)}) = \beta ((1 - \tau) y_{i,t})^{-\rho} + (1 - \beta) E_t^{-\rho} \tag{27}
\]

while utility under private education is given by:

\[
U_r(y_{i,t} = \frac{BS_t}{(1 - \tau)}) = \beta \left( \frac{B ((1 - \tau) y_{i,t} + S_t)}{(1 + B)} \right)^{-\rho} + (1 - \beta) \left( \frac{(1 - \tau) y_{i,t} + S_t}{(1 + B)} \right)^{-\rho} \tag{28}
\]

Since it is assumed \(S_t < E_t\), \(U_u(y_{i,t} = \frac{BS_t}{(1 - \tau)}) > U_r(y_{i,t} = \frac{BS_t}{(1 - \tau)})\). A household with income \(y_{i,t} = \frac{BS_t}{(1 - \tau)}\) will prefer public education over private education.

The second step is to show that there exists a level of income where \(U_u < U_r\). To show this consider the case where \(S_t = 0\). If the result can be obtained for this case, then it will hold more generally when \(S_t > 0\). Utility under public education is given by:

\[
U_u(y_{i,t}) = \beta ((1 - \tau) y_{i,t})^{-\rho} + (1 - \beta) E_t^{-\rho} \tag{29}
\]

while utility under private education is given by:

\[
U_r(y_{i,t}) = \beta \left( \frac{B (1 - \tau) y_{i,t}}{(1 + B)} \right)^{-\rho} + (1 - \beta) \left( \frac{(1 - \tau) y_{i,t}}{(1 + B)} \right)^{-\rho} \tag{30}
\]

Substituting (29) and (30) into the inequality \(U_u < U_r\) and manipulating yields the following threshold income:

\[
y^+ = \frac{E_t}{(1 - \tau)} \left( (1 + B)^{\rho+1} - B^{\rho+1} \right)^{\frac{1}{\rho}} > \frac{BE_t}{(1 - \tau)} > \frac{BS_t}{(1 - \tau)} \tag{31}
\]
With no voucher, $S_t = 0$, a household with income $y_{i,t} = y^+$ will be indifferent between public and private education. If we give this household a private education voucher $S_t > 0$ utility under private education will increase while utility under public education will not change so private education is preferred to public education. We can also infer that if $y_{i,t} > y^+$, $U^r(y_{i,t}) > U^u(y_{i,t})$. As a result, $U^r$ and $U^u$ will cross at some income between $\frac{BS_t}{(1-\tau)}$ and $y^+$.

The third step is to show that this crossing will be unique. Consider an income level where households are indifferent between public and private education, $y^*$. Any increase in income above $y^*$ will increase both $U^r$ and $U^u$. However, the increase $U^r$ must be greater than the increase in $U^u$, since the allocation decision in public education is constrained (households cannot spend any of the extra income on education if they stay in the public system), while the allocation decision in private education is unconstrained. This has some similarities to the Le Chatelier Principle. Thus, if income exceeds this threshold, the household will prefer private education. This shows that any income $y^*$ where $U^r = U^u$ must be unique. Taken together, this proves Proposition 2.
Figure 1: Sensitivity Analysis-Production Parameters: Case A with $\theta = 1.86, \gamma = 0.01, \rho = 0.54, \beta = 0.967$. Time series of per-capita, poorest family’s and richest family’s human wealth, (a), (b) and (c) respectively. Growth rates for per capita human wealth, (d). Human wealth inequality, measured by Gini coefficient, (e), and by ratios of ninth to first decile income, (f). Education expenditure, including all vouchers, (g), per student public education expenditure, (h), and public education enrollments, (i). Ratios of the voucher cases to the no voucher case in panels (a) to (c) and (e) to (h), with $S_0 = $12000 (solid curve) and $S_0 = $24000 (dashed curve). In panels (d) and (i), plots are for the cases $S_0 = $0 (solid curve), $S_0 = $12000 (dashed curve) and $S_0 = $24000 (short dashed curve).
Figure 2: Sensitivity Analysis-Production Parameters: Case A with $\theta = 2.99, \gamma = 0.20, \rho = 0.54, \beta = 0.967$. Time series of per-capita, poorest family's and richest family's human wealth, (a), (b) and (c) respectively. Growth rates for per capita human wealth, (d). Human wealth inequality, measured by Gini coefficient, (e), and by ratios of ninth to first decile income, (f). Education expenditure, including all vouchers, (g), per student public education expenditure, (h), and public education enrollments, (i). Ratios of the voucher cases to the no voucher case in panels (a) to (c) and (e) to (h), with $S_0 = $12000 (solid curve) and, $S_0 = $24000 (dashed curve). In panels (d) and (i), plots are for the cases $S_0 = $0 (solid curve), $S_0 = $12000 (dashed curve) and $S_0 = $24000 (short dashed curve).
Figure 3: Sensitivity Analysis-Production Parameters: Case B with $\theta = 1.86, \gamma = 0.01, \rho = -0.359, \beta = 0.750$. Time series of per-capita, poorest family’s and richest family’s human wealth, (a), (b) and (c) respectively. Growth rates for per capita human wealth, (d). Human wealth inequality, measured by Gini coefficient, (e), and by ratios of ninth to first decile income, (f). Education expenditure, including all vouchers, (g), per student public education expenditure, (h), and public education enrollments, (i). Ratios of the voucher cases to the no voucher case in panels (a) to (c) and (e) to (h), with $S_0 = $12000 (solid curve) and, $S_0 = $24000 (dashed curve). In panels (d) and (i), plots are for the cases $S_0 = $0 (solid curve), $S_0 = $12000 (dashed curve) and $S_0 = $24000 (short dashed curve).
Figure 4: Sensitivity Analysis-Production Parameters: Case B with $\theta = 2.89, \gamma = 0.20, \rho = -0.359, \beta = 0.750$. Time series of per-capita, poorest family’s and richest family’s human wealth, (a), (b) and (c) respectively. Growth rates for per capita human wealth, (d). Human wealth inequality, measured by Gini coefficient, (e), and by ratios of ninth to first decile income, (f). Education expenditure, including all vouchers, (g), per student public education expenditure, (h), and public education enrollments, (i). Ratios of the voucher cases to the no voucher case in panels (a) to (c) and (e) to (h), with $S_0 = $12000 (solid curve) and, $S_0 = $24000 (dashed curve). In panels (d) and (i), plots are for the cases $S_0 = $0 (solid curve), $S_0 = $12000 (dashed curve) and $S_0 = $24000 (short dashed curve).
Figure 5: Sensitivity analysis-Preference Parameters. Parameter values, $\theta = 2.34, \gamma = 0.1, \rho = 1.00, \beta = 0.991$. Time series of per-capita, poorest family's and richest family's human wealth, (a), (b) and (c) respectively. Growth rates for per capita human wealth, (d). Human wealth inequality, measured by Gini coefficient, (e), and by ratios of ninth to first decile income, (f). Education expenditure, including all vouchers, (g), per student public education expenditure, (h), and public education enrollments, (i). Ratios of the voucher cases to the no voucher case in panels (a) to (c) and (e) to (h), with $S_0 = $12000 (solid curve) and $S_0 = $24000 (dashed curve). In panels (d) and (i), plots are for the cases $S_0 = $0 (solid curve), $S_0 = $12000 (dashed curve) and $S_0 = $24000 (short dashed curve).
Figure 6: Sensitivity analysis-Preference Parameters: Parameter values, $\theta = 2.275, \gamma = 0.1, \rho = -0.70, \beta = 0.590$. Time series of per-capita, poorest family's and richest family's human wealth, (a), (b) and (c) respectively. Growth rates for per capita human wealth, (d). Human wealth inequality, measured by Gini coefficient, (e), and by ratios of ninth to first decile income, (f). Education expenditure, including all vouchers, (g), per student public education expenditure, (h), and public education enrollments, (i). Ratios of the voucher cases to the no voucher case in panels (a) to (c) and (e) to (h), with $S_0 = $12000 (solid curve) and $S_0 = $24000 (dashed curve). In panels (d) and (i), plots are for the cases $S_0 = $0 (solid curve), $S_0 = $12000 (dashed curve) and $S_0 = $24000 (short dashed curve).