Thermal Processes in the Solar System

Astronomy Decoded

Lecture 3

3.1 Composition and Differentiation of Solar System

Basic rock types classified according to history:

Primitive rock - not on Earth or Moon (e.g. carbonaceous chondrites).
Igneous rock - cooled molten material.
Sedimentary rock - deposited rock, organism fragments.
Metamorphic rock - other rock type transformed at high temperature and pressure.

Density of common material: Metals ~ 8 $g\ cm^{-3}$
Rocks ~ 2.5 - 3.5 $g\ cm^{-3}$
Ices ~ 1.0 $g\ cm^{-3}$

T = Terrestrial, **J** = Jovian, **P** = Primitive:

ρ_{Earth} = 5.52 $g\ cm^{-3}$, **T** with rock and large iron-nickel metal core.
$\rho_{Mercury}$ = 5.43 $g\ cm^{-3}$, **T** with disproportionately large iron core.
ρ_{Venus} = 5.25 $g\ cm^{-3}$, **T** with rock and large metal core.
ρ_{Mars} = 3.95 $g\ cm^{-3}$, **T** with rock and iron deficiency.
ρ_{Moon} = 3.34 $g\ cm^{-3}$, **T** mostly rock with small metal core at most (ice!)
ρ_{IDA} = 2.2-2.9 $g\ cm^{-3}$, **P** asteroids may be remnants of "warm" planetesimals, and have densities varying from that of rocks to metals.
$\rho_{Meteoroids} \approx \rho_{Asteroids}$, **P** fragments of asteroids, different kinds with different densities.
ρ_{Pluto} = 2.03 $g\ cm^{-3}$, **P** rock and ice, possible the remnant of a large, "cold", planetesimal.

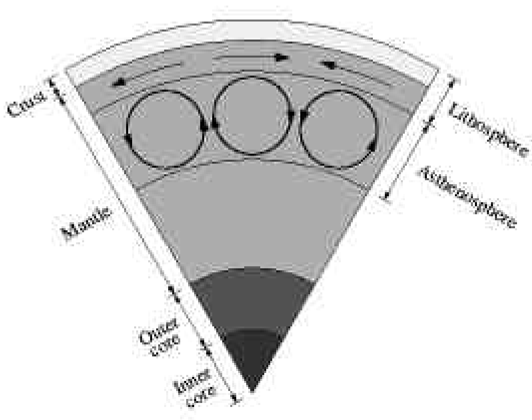


Fig. 19.10: The interior structure of the Earth.

T = Terrestrial, **J** = Jovian, **P** = Primitive:

$\rho_{Neptune}$ = 1.64 *g cm*$^{-3}$, **J** gas envelope, liquid metal interior, and rock core.
$\rho_{Jupiter}$ = 1.33 *g cm*$^{-3}$, **J** gas envelope, liquid metal interior, rock (and ice?) core.
ρ_{Mimas} = 1.2 *g cm*$^{-3}$, **X** outer satellite of Saturn, mostly ice.
ρ_{Saturn} = 0.69 *g cm*$^{-3}$, **J** large gaseous envelope.
ρ_{Comets} = 0.1-1.0 *g cm*$^{-3}$, **P** dirty smowballs, possibly the remnants of small, "cold" planetesimals.

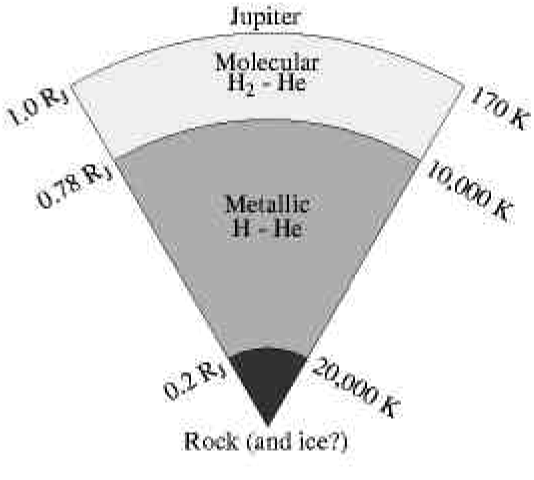


Fig. 20.5: (a) Computer model of Jupiter's interior.

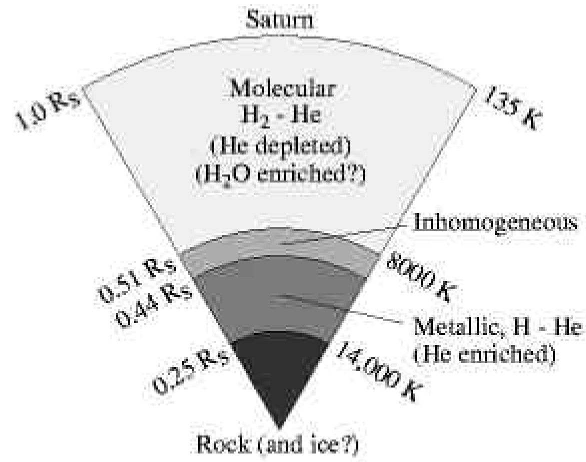


Fig. 20.5: (a) Computer model of Saturn's interior.

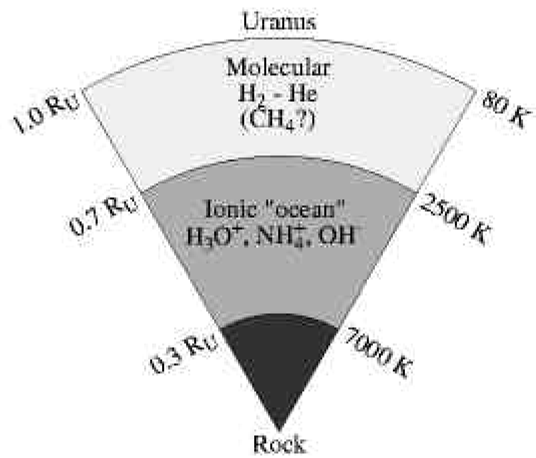


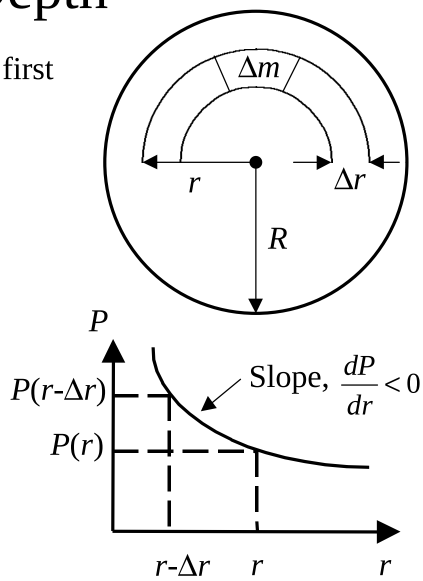
Fig. 20.5: (a) Computer model of Uranus' interior

3.2 Internal Pressure Variation with Depth

Assume homogenous structure for first approximation.
For mass element $\Delta m \approx r(A\,\Delta r)$:

Net pressure force outwards:

$$= [P(r-\Delta r) - P(r)]A$$
$$= [P(r) + \Delta P - P(r)]A$$
$$= A\Delta P$$



Weight = $\Delta m g = r(A\Delta r)g$

$\therefore$ if Δm is in "hydrostatic equilibrium",

$$A\Delta P = rA\Delta r g$$

$$\Rightarrow \frac{\Delta P}{\Delta r} = rg$$

$$\therefore \frac{dP}{dr} = -rg, \text{ since } P \text{ decreases as } r \text{ increases.}$$

We will use a very rough approximation $r(r) \approx$ constant

$$\Rightarrow \int_R^r \frac{dP}{dr}dr = -r\int_R^r g\,dr, \text{ where } g = \frac{GM(r)}{r^2}$$

$$\Rightarrow g = \frac{Gr\left(\frac{4}{3}pr^3\right)}{r^2} = \frac{4p}{3}Grr; \text{ (Vol of Sphere } = \frac{4}{3}pr^3)$$

$$\int_R^r dP = -r\int_R^r \frac{4p}{3} G\, r\, r dr = \frac{-4p G r^2}{3}\int_R^r r dr$$

$$\Rightarrow P(r) = \frac{2p}{3} G r^2 \left(R^2 - r^2\right)$$

$$\therefore \text{ At core where } r = 0, P(0) = \frac{2p}{3} G r^2 R^2$$

For Earth, $P(0) = 1.73 \times 10^{11}\, Nm^{-2}$

For Moon,

$$P(0) = \left(\frac{r_{\text{Moon}}}{r_{\text{Earth}}}\right)^2 \left(\frac{R_{\text{Moon}}}{R_{\text{Earth}}}\right)^2 P_{\text{Earth}}(0)$$

$$= (0.61)^2 \times (0.27)^2 \times 1.73 \times 10^{11}$$

$$= 4.7 \times 10^9\, Nm^{-2}$$

Rocks and iron crushed by pressures > 4×10^8 Nm^{-2}

Seismic waves passing through the Earth's interior indicate an outer core of liquid metal. Temp must be ~ melting temperature of iron. (~1500^{o}K at 1 atmosphere, but ~4000^{o}K at core pressures)

Heat flow out from core ~0.06 Jm^{-2} at Earth's surface.
What causes these high temperatures?

3.3 Differentiation Model

• Planets start to condense out of “solar system” surronding young Sun.
• Material accretes onto forming planets (~4.6 billion years ago).
• Planetary interiors heat up due to:
  • kinetic energy of accreted material.
  • deformation due to major impacts.
  • primary radioactive decay.

∴ Solar system bodies large enough should melt and form a sphere.

⇒ Dense material settles to core.
⇒ Surface cools and forms brittle crust.

Evidence: All solar system bodies with d>~ 200 *km* are approx spherical. Most solar system bodies with d~< 130 *km* are irregular in shape.

The Martian moon Phobos.

Global image of Oberon synthesised from Voyager II data.

3.4 Structure and Differentiation of Planetary Atmospheres

This time assume hydrostatic equilibrium,

$\frac{dP}{dr} = -rg$ again, and the rough

approx $g(r) = $ constant.

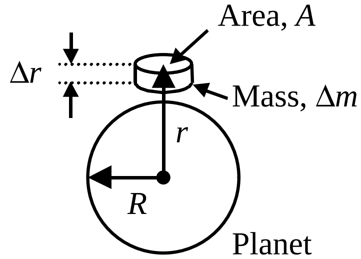
3.4.1 Estimate of Atmospheric Mass

$$\Delta m = rA\Delta r \Rightarrow \frac{dm}{dr} = rA = \left(-\frac{1}{g}\frac{dP}{dr}\right)A$$

$$\Rightarrow \int \frac{dm}{dr}dr = -\frac{A}{g}\int \frac{dP}{dr}dr$$

$$\Rightarrow \int_0^m dm = \frac{-A}{g}\int_{P(R)}^{P(r)} dP$$

$$\therefore m = \frac{-A}{g}\left[P(r) - P(R)\right]$$



For total atmosphere, pressure at top, $P(r) \approx 0$

$$\therefore m = \frac{A}{g}P(R) = \frac{4pR^2}{g}P(R)$$

For Earth, $m \approx 5.3\times 10^{18}\,kg$

3.4.2 Pressure Variation with Height

Assume ≈ Ideal Gas, $PV = nkT \Rightarrow P = \frac{n}{V}kT$

$$\Rightarrow P = \frac{r}{m}kT \Rightarrow r = \frac{mP}{kT};\ (m = \text{molecular mass})$$

Hydrostatic equilibrium, $\frac{dP}{dr} = -rg$ & $r = \frac{mP}{kT}$

$$\Rightarrow \frac{dP}{dr} = -\frac{mg}{kT}P$$

Over relatively short distances, $g(r)$ & $T(r) \approx$ constant

$$\Rightarrow \int_R^r \frac{1}{P}\frac{dP}{dr}dr \approx -\frac{mg}{kT}\int_R^r dr$$

$$\Rightarrow \ln\frac{P(r)}{P(R)} = -\frac{mg}{kT}(r-R)$$

but $r - R = h$, the height above the planet's surface.

$$\therefore P(r) \approx P(R)e^{-\frac{mg}{kT}h} = P(R)e^{-\frac{h}{H}}$$

$\Rightarrow$ "Barometric equation" with H = scale height = $\dfrac{kT}{mg}$

Large *m* gives small *H* so heavy molecules are concentrated at lower altitudes.
Hence differentiation of constituents.

3.4.3 Differential Escape of Atmospheric Gases

If $E_{total} = K + U > 0$ then all of a planet's atmopshere will be quickly lost (e.g. the Moon).

But even if $E_{total} < 0$ on average, collisions will speed some molecules to $v > v_{esc}$ then that atmospheric component will be lost by now.

Statistical Mechanics can be used to estimate atmospheric lifetimes.

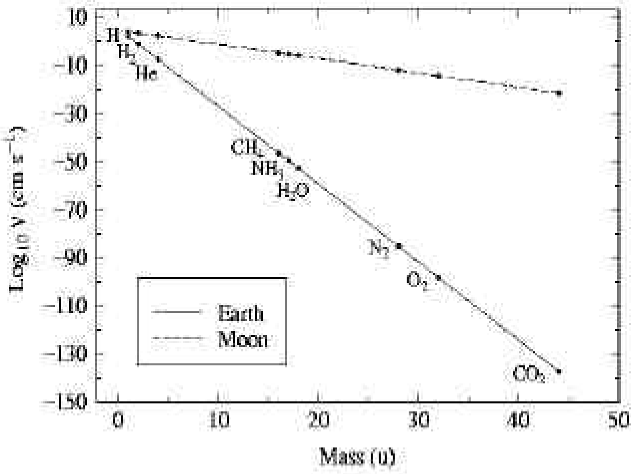


Fig. 18.6: Log(atmospheric escape parameter) versus atomic weight.

Viking I lander site.