Philosophy 2/3 FOL

Tutorial Sheet 6.
Technical Developments.

Concepts.

Mathematical induction, formula induction, proof induction - base case and induction step.

Soundness and completeness.

Exercises.

1. Prove by mathematical induction (formula induction) that any 4-rowed truth-table for any formula A with just the connectives ~ and ≡ must have an even number of T's and F's.
[G. Hunter, *Metalogic, An Introduction to the Metatheory of Standard First-Order Logic*, Uni. of California Press, 1973, pp.89-90.]
[Hint: Determine the shape of the formula for the base case. There are 2 induction steps that need to be reasoned through.] Show that □(□p ⊃ p) is a logical truth, just using the semantics of N□.

2. In the Completeness Lemma for classical logic, we let b be an open complete branch and let v be the interpretation induced by b. In the proof by formula induction, we show for each of the tableau rules that (i) if A is on b then v(A) = 1, and (ii) if ~A is on b then v(A) = 0. Prove this for the ≡- and ~≡-rules.

3. In the Soundness Lemma for basic modal logic K, we let I be an interpretation faithful to an initial list of formulae. In the proof by induction on proof steps, we show for each of the tableau rules that I remains faithful to at least one of the extensions of the branch involved. Show this for the ~□- and v-rules.