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Statistical Mechanics, Soliton Theory, and Nonlinear Dynamics

In honour of H.W. Capel
on the occasion of his 60th birthday

Editors:

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AMSTERDAM – LAUSANNE – NEW YORK
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In honour of H.W. Capel
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Preface

To Hans Capel on the occasion of his 60th birthday

1. Introduction

This Festschrift has been put together to honour Hans Capel's extraordinarily active career in research and his enormous contribution to theoretical physics. We celebrate his large number of important contributions to a wide variety of fields as well as his inspiring personality and his enormous ability to motivate young researchers to pursue active research. Those who have had the pleasure of working with him had the opportunity to benefit from his excellent taste for nice problems as well as from his deep insights in physics. His flexibility of mind has allowed him time and again to pick up new subjects, delving into new unfamiliar areas and steering right into the heart of the problem. His good humour, as well as his deep-rooted care for his students and colleagues, have made him a personal friend to many and a scientific father-figure to his students.

Most colleagues of Hans Capel probably know him only by the one particular aspect of his work that they happen to share with him. They might not always be in a position to appreciate the "other sides" of Hans, which are many. Glancing over his list of publications[1] might give the impression that here is an extraordinary person that cannot be tied to one single scientific habitat, a scientific chameleon that has changed his colours several times during his career. For somebody who doesn't know Hans personally, it is probably quite difficult to see the links between the various subject areas that Hans has been actively contributing to. Nonetheless, for those who know Hans a bit better there is a clear and logical line connecting the various areas of his interest. In this Editorial we want to give a short overview of Hans' work and explain how the different areas are interconnected.

Very roughly speaking, Hans Capel's interests can be divided into three broad areas: (i) statistical mechanics, (ii) soliton theory and integrable systems and (iii) chaos and dynamical systems. It was the original idea of the Editors to organise the contributions to this Festschrift according to these three areas of research. There are, however, a number of contributions that are situated on the borderlines between these areas.

[1] A selection of Capel's publication list is contained in the references below.

2. Statistical mechanics

Hans Willem Capel started his research with the theoretical study of fundamental properties of complicated magnetic systems, such as rare earth compounds [1]. These studies led him to consider the spin-one Ising model with crystal-field splitting within the molecular-field approximation, which exhibits a tricritical point where the phase transition changes from second order to first order [2,3]. This model is now widely known as the Blume–Capel model.

During his stay at the University of British Columbia, Capel worked on the De Haas–Van Alphen effect, the broadening of Landau levels [4,5], topics that are of much current interest in the solid-state community. He continued by investigating magnetic breakdown and energy-band structures [6] and started to worry about questions of how much of an effect is due to the model and how much is due to the approximations. In this way, he was led to the investigation of exactly-solvable models during his many fruitful years at Leiden.

At that time, there were many equivalent-neighbour models being solved and, together with Tindemans, Capel started to search for a general framework to treat these models [7], using saddle point techniques to study cases with both attractive and repulsive interactions. N.N. Bogoliubov Jr. [8] had devised an alternative approach based on rigorous inequalities leading to a min max principle. A sequence of papers [9–12] followed, simplifying these results and generalizing them to the case of systems with short- and long-range interactions whose Hamiltonians are smooth functions of a finite set of short-range operators. A first application of the general formalism was to the problem of stability of (multi)critical behaviour [13–15] under perturbations of the system by min max constraints as can be caused, for example, by the addition of such extremely long-range interactions. A second application was to a class of models for liquid Helium-3 [16–19], deriving Landau expansions and phase diagrams.

In the early seventies Mazur and Siskens studied another class of exactly solvable models, the XY chain, and it was natural for Capel to get involved also. First, this led to a series of papers on high-temperature expansions and exact results for susceptibilities in XY, Heisenberg, and Ising chains [20–23]. A seminar by Siskens on the XY model with Dzyaloshinsky interactions [24] brought one of us (JHHP) to Leiden [25]. A detailed study of the susceptibility of the XY chain with alternating interactions [26] followed, which led to a different interpretation of experiments that were being done in Leiden at that time [27].

In 1975 Sur, Jasnow, and Lowe made the amazing conjecture [28] that the transverse autocorrelation function of the isotropic XY-model at infinite temperature is a pure Gaussian. This was proved and generalized in a number of papers [29–34], relating these correlation functions to solutions of classical integrable equations of motion, such as the Toda lattice. Later it was shown that the underlying Wick theorem could be further extended leading to integrable equations for all temperatures [35,36] and the general Z-invariant Ising model of Baxter [37].

Capel was always taking time to interact with the experimental groups at Leiden, even to the point of joint publications [38–42], where the first work involves a spin $\frac{1}{2}$ system and the last few works involve spin-wave treatments, as several of the experiments involve effective spin $\frac{3}{2}$ or higher systems. Capel also remained interested in spin 1 systems [43].

Finally, all the above developments naturally led to a further study of integrable models for classical Heisenberg chains [44]. This is discussed in more detail in the next section.

This section of the Festschrift has a number of contributions. First, the exactly solvable models are represented by a paper on interacting dimers on the honeycomb lattice by Huang, Wu, Kunz and Kim, who discuss a certain five-vertex singular limit of the six-vertex model in more detail. Berkovich, McCoy and Schilling show how $N = 2$ supersymmetric conformal field theory characters satisfy new Rogers–Ramanujan identities related to Bailey pairs. Warnaar, O'Brien and Pearce present results on the conformal spectra of the Ising model with fixed boundaries. Au-Yang and Perk discuss how much one may learn about phase diagrams in generalized (chiral) clock models, using existing exact results. Next, there is a paper on the effective action for the superfluid Fermi-gas by Brussaard and van Weert, continuing the research on Helium-3. Finally, Mukhin and De Jongh are contributing a paper on the t–J model, obtaining results for the quasi-particle dispersion, Fermi surface and optical conductivity.

3. Soliton systems and integrability

In the course of 1979–1982 Hans Capel's interests shifted slightly in other directions. The reason for this shift was the arrival of two new PhD students (the undersigned GRWQ and FWN). With the first, Hans Capel initiated a project on the dynamics of magnetic spin chains, which formed a natural sequel to the work that was mentioned in the previous section on the XY model. With the latter, a project was initiated to investigate the phase diagram of superfluid Helium 3 in the presence of a magnetic field. This was a continuation of the work that was also mentioned in the previous section on rigorous approaches for calculating free energies of extreme long-range interaction systems in statistical mechanics. The work on Helium 3 was mentioned briefly already in the previous section as well. Here we will focus on how the work on the dynamics of spin chains finally grew into a large body of work on soliton systems, exactly integrable hierarchies, direct linearisation of such systems and the development of a whole new theory of discrete- and lattice integrable systems.

The project started with some interesting results obtained in papers [44,45] on the classical Heisenberg spin chain equation, in particular leading to Miura transformations between the equation of motion for the anisotropic Heisenberg ferromagnet (with uniaxial anisotropy) and equations of nonlinear Schrödinger type. These papers focused also on special solutions of soliton and similarity type, thus making connections with the Painlevé II and IV equations and equations classified by Chazy at the beginning of this century. When FWN joined in this work as a side-exercise to his own PhD project, the emphasis gradually shifted towards a systematic approach, which — following an article by Fokas and Ablowitz, [47] — was baptised "Direct Linearisation". This approach started from linear singular integral equations, for example an equation of the form, [48],

$$\varphi_k + \int\limits_{C^*} d\lambda^*(\ell') \int\limits_{C} d\lambda(\ell) \frac{\rho_k \rho^*_{\ell'} \varphi_\ell}{(k-\ell')(\ell'-\ell)} = \rho_k c_k \ , \tag{3.1}$$

which were found based on a treatment by Rosales, [46]. This was the beginning of a very exciting and enjoyable time, where Hans Capel and his two students, with the assistance of Dr. J. van der Linden, formed a small group at the Lorentz Institute of the University of Leiden, exploring the new area of soliton physics and learning new and beautiful mathematics. It was a very productive period in which many papers were written, cf. e.g. [49–57], since Hans and his students were able to develop a novel point of view on soliton systems which turned out to be very fruitful. This point of view that came out of the Direct Linearisation approach led to many new results. One particular aspect of the approach was a description in terms of infinite matrices which turned out to be a very flexible scheme to provide connections between the various soliton systems and thus to obtain new insights in the underlying structures, cf. [49,54]. One of the main developments was a gradual change of point of view moving from continuous systems (integrable nonlinear evolution equations), [48–52], to semi-discrete systems (integrable differential-difference equations), [53], and finally to the fully discrete equations (lattice equations, partial difference equations), [55–65]. Going from continuous to discrete was a gradual growth process, but actually also a reversal in terms of simplicity. Ultimately, Hans Capel and his students realised that the discrete systems basically contained everything there was to know about integrability. The beauty of these discrete systems is that they are nothing but the Bianchi identities for the permutability of Bäcklund transformations, [50–58], for which the Direct Linearisation Approach gave a very neat description, [51]. Another realisation that followed from this approach, was that all these systems (continuous, semi-discrete and fully discrete) are all compatible with each other. Of course Hans Capel's group was not the only one to investigate discrete integrability. Earlier results, notably by Ablowitz and Ladik and by Hirota, [59,60], already existed. But none of these approaches had the overwhelming systematics of the Direct Linearisation Approach, which provided the various interconnections between discrete and continuous equations. A particular example of a new result that was discovered in this way was the following universal integrable lattice equation [55]:

$$\begin{aligned} &\frac{1 - (p+\beta)s_{n+1,m} + (p-\alpha)s_{n,m}}{1 - (q+\beta)s_{n,m+1} + (q-\alpha)s_{n,m}} \\ &\quad = \frac{1 - (q+\alpha)s_{n+1,m+1} + (q-\beta)s_{n+1,m}}{1 - (p+\alpha)s_{n+1,m+1} + (p-\beta)s_{n,m+1}} \ , \end{aligned} \tag{3.2}$$

which, by performing different continuum limits, can be shown to be related to a host of different equations: the KdV and the MKdV equations, and their discrete counterparts, the Toda lattice, the Volterra system, etc. It was recently realised, cf. e.g. [61], that eq. (3.2) is actually a discrete version of the so-called Schwarzian KdV equation which is the most fundamental of all KdV related equations, and it is this equation that is starting to appear in recent studies in connection with discrete complex analysis and conformal field theory.

Other results include generalisations of lattice systems to higher dimensions, [62–65], Hamiltonian aspects, [64], reductions to integrable mappings, [66–68], and quantisation of lattice systems and mappings, [69–71].

Hans Capel is still very active in the domain of soliton systems, and he has always kept alive the collaboration with his former PhD-students with whom he has been working on integrable systems. The investigations in integrable discrete systems have had many far-reaching consequences. They have initiated new lines of research, including: integrable multi-dimensional mappings, discrete Painlevé equations, symmetry approaches of difference equations and exact quantisation methods for discrete systems. The research in discrete systems is very active now and becoming more and more of central interest in integrable systems. As early as the beginning of the eighties Hans had the almost prophetic insight that this was going to be an important domain for the future.

There are seven contributions to this part of the Festschrift. De Jager et al. investigate the geometric nature of Bäcklund transformations. This is very appropriate because of Hans' own interests in these structures. Hans Capel's interests in discrete integrable systems are well-represented: The contribution of O. Ragnisco and M. Bruschi deals with some new finite-dimensional integrable systems. A. Ramani and B. Grammaticos, in their article, deal with discrete versions of Painlevé equations and the contribution by V. Papageorgiou and FWN presents some new higher-order integrable discrete systems. B. Fuchssteiner in his article investigates new particle structures in integrable evolution equations. Finally, with the contributions of M. Ablowitz et al. and by A.S. Fokas and T. Bountis we are already at the borderline between integrability and chaos, a border that Hans himself has crossed also. These contributions clearly show that, in accordance with what Hans passionately believes, the theory of chaos and the theory of integrable systems cannot be separated as two distinct subject areas. Chaos on the one hand and integrability on the other hand, are undeniably linked together as two sides of one and the same coin, and Hans is one of those very rare people who have explored both sides at a very deep level.

4. Nonlinear dynamics and chaos

Hans Capel's first contributions to chaos theory concerned the crossover from conservative to dissipative behaviour in very weakly dissipative dynamical systems undergoing period doubling [72–74]. This study, in collaboration with van der Weele, was inspired by the work of Zisook and by a talk by Ghendrih at one of the famous Dynamics Days conferences at Twente University [75,76]. These investigations by Capel and van der Weele culminated in a more accurate determination of the universal crossover function $\delta_{2,\mathrm{uni}}(B)$.

One-dimensional dynamical systems have been a second fruitful area of research for Capel and his collaborators. They first investigated the dependence of the Feigenbaum scaling factors α and δ on the order z of the maximum of the map [77,78]. In particular they carried out a careful numerical study of the behaviour of $\alpha(z)$ and $\delta(z)$ as z gets very large.

This work was followed by a nice study on intermittency and window scaling in one-dimensional maps [79,80].

Another one-dimensional topic, that was studied in collaboration with Kluiving and Pasmanter, was piecewise linear maps [81–83]. This collaboration eventually led to joint work on hydrodynamical turbulence (see the appendix to the paper by Kluiving and Pasmanter in this volume).

Fractals are of course well known as a paradigm of the nonlinear scientific revolution. In a series of papers with Milosevic and his group [84,85], Capel studied directed random walks on fractals, thus merging fractals with his longstanding interest in statistical mechanics.

Finally, a major strand of Capel's chaos research is on dynamical systems possessing time-reversal symmetry. These systems provide a link between two of Capel's main scientific interests: integrable dynamical systems on the one hand, and chaotic dynamical systems on the other. Most integrable systems possess (generalised) time-reversal symmetry [86,87], so it is natural to ask what happens when such systems are perturbed in such a way that they are no longer integrable, but still do possess time-reversal symmetry. The answer turns out to be that the perturbed systems are hybrids exhibiting both conservative and dissipative phenomena [88,89].

This section of the Festschrift contains five contributions. Van der Weele and de Kleine study the motion of a spinning pendulum as a function of its two control parameters. It is shown that, for different values of the parameters, this system exhibits either predominantly chaotic or predominantly regular behaviour. Kluiving and Pasmanter introduce a new model for inertial-subrange turbulence, which is compared with experimental observations. The paper by J.A.G. Roberts investigates the dynamics of trace maps, a subject intimately connected with the theory of 1-D quasicrystals. Christov and Nicolis derive a coupled map lattice from the φ^4 equation, and show that it exhibits spatio-temporal chaos and strong intermittency. Finally, J.S.W. Lamb's contribution shows that the investigation of the similarities and differences between conservative systems and systems with time-reversal symmetry (a topic studied by Capel in the past [90]), is still an active area of research.

In conclusion, we hope that this volume will illustrate the breadth of Hans Capel's research interests as well as the affection of his students and scientific friends. We wish Hans Capel many more years of active research.

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