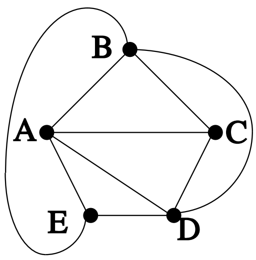
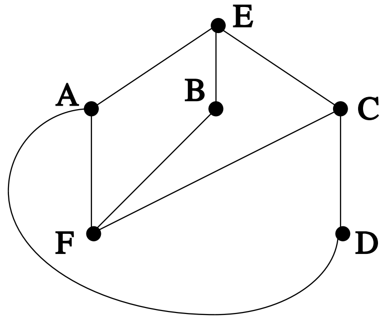
1. (a) This graph has more than two vertices of odd degree so there can be no Eulerian Path nor an Eulerian Circuit.
   (b) This graph has exactly two vertices of odd degree so there is an Eulerian Path but no Eulerian Circuit.
   (c) This graph has all vertices of even degree so there is an Eulerian Circuit.
   (d) This graph has exactly two vertices of odd degree so there is an Eulerian Path but no Eulerian Circuit.
   (e) This graph has all vertices of even degree so there is an Eulerian Circuit.
   (f) This graph has all vertices of even degree so there is an Eulerian Circuit.

2. (a) Sequence of edges: ba ad db bc cd de ea af fb be ec cf fe
   Sequence of vertices: b a d b c d e a f b e c f e
   (b) Sequence of edges: AB BC CA AE EC CD DB BF FD DE EF FA
   Sequence of vertices: A B C A E C D B F D E F A
   (c) When edge FA is removed the graph becomes disconnected, ie, FA is a bridge.

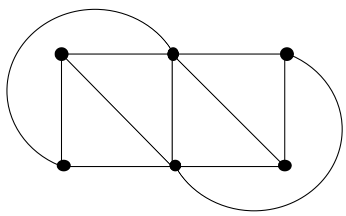
3. (a)



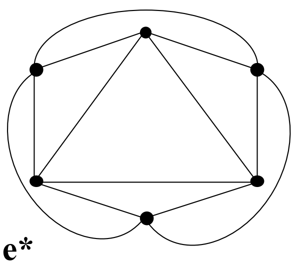
(b)



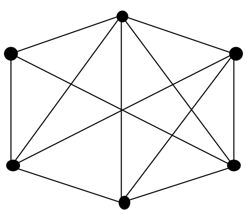
4. Graph **1a** is isomorphic to graph **1a*** and graph **1e** is isomorphic to graph **1e***. This means **1a** and **1e** are planar graphs.



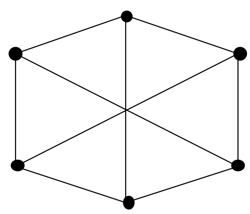
1a*



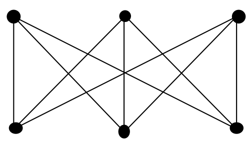
1e*



1d



1d*



$K_{3,3}$

Graph **1d** contains a copy of **1d***, and graph **1d*** is isomorphic to the graph $K_{3,3}$. This means **1d** is a non-planar graph.

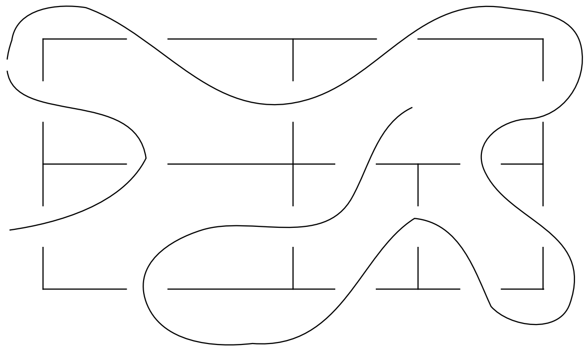
5.

POLYHEDRON matches with PLANE GRAPH	a	b	c	d	e	f	g	h	i
	......	...	...	...	...	......	......	...	...
	ii & vi & x	iv	i	i	vii	v & viii	iii	ix	ix

6. Using Euler's Theorem for Paths and Circuits, and applying it to the corresponding plane graphs, travelling along each edge exactly once is possible for polyhedron **e** and for polyhedron **g**, but not for any of the other polyhedra.

   Polyhedron **e** is known as an octohedron, and an Eulerian circuit would be produced. Polyhedron **g** has two vertices of odd degree and an Eulerian path would be possible, but not an Eulerian circuit.

7. Every time we enter a room and then leave again we walk through exactly two doors. If a room has an odd number of doors, then the only way to walk through each of its doors exactly once is to either start our walk or end our walk in that room. Thus there must be at most two rooms which have an odd number of doors. The figure below shows that such a walk is possible for House (a). In House (b) no such walk is possible since there are three rooms which have an odd number of doors.



   Alternatively we may model the problems by graphs by placing a vertex " • " in every room and if there is an odd number of external doors, place a single vertex in the outside region. Then make a connection between adjacent rooms using internal and external doors. By noting the degree of the vertices we may apply the test for the existence of Eulerian circuits, Eulerian paths, or whether no Eulerian path exists. On the left below, there are exactly 2 vertices of odd degree, so it follows that an Eulerian path is possible. Fleury's Algorithm will provide a path. On the right below, there are 4 vertices of odd degree, so it follows that no Eulerian path is possible.