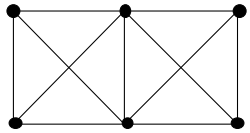
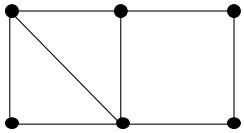
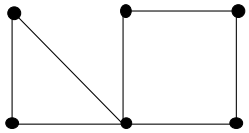
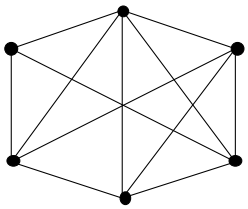
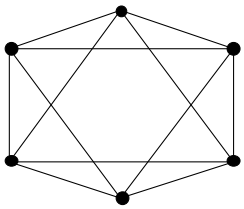
1. Write down the degree of each vertex beside each vertex for each of the following six graphs. Use Euler's Theorem on Paths and Circuits to decide which graphs contain an Eulerian Circuit, which graphs contain an Eulerian Path (but no Eulerian Circuit) and which graphs have neither an Eulerian Circuit nor an Eulerian Path.

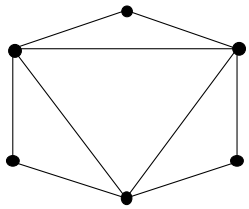
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

Fleury's Algorithm, simplified version
If there are exactly two vertices of odd degree, start at one on them.
From any possible vertices to start, choose the one whose label occurs first in the alphabet.
If there is a choice of edges at some step, then choose the one that travels to the vertex with the alphabetically earliest label.
Only cross a bridge if there is no alternative.
Write down each vertex as it is visited.

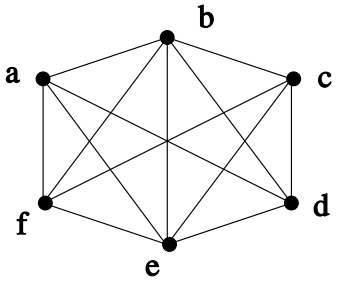
2. (a) For the graph on the left below, apply Fleury's Algorithm to make an Eulerian Path or Eulerian Circuit.

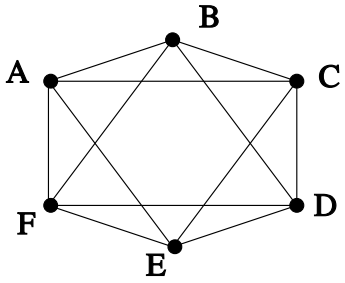
   (b) For the graph in the centre, apply Fleury's Algorithm to make an Eulerian Path or Eulerian Circuit.

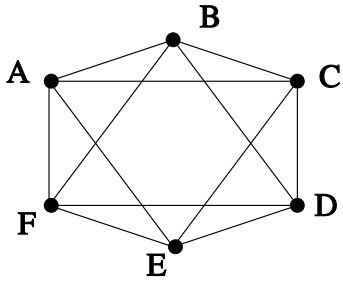
   (c) For the graph on the right below, cross out the edges in the following order.

   AB,BC,CD,DE,EF,FA,AC,CE,EA

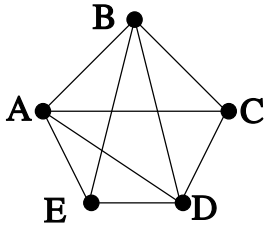
   Notice that further progress is impossible. Why is there a failure to create an Eulerian Path or Eulerian Circuit ?

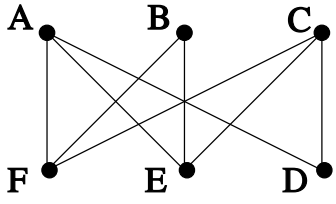






3. In each case, show that the graph is planar by drawing an isomorphic plane graph.

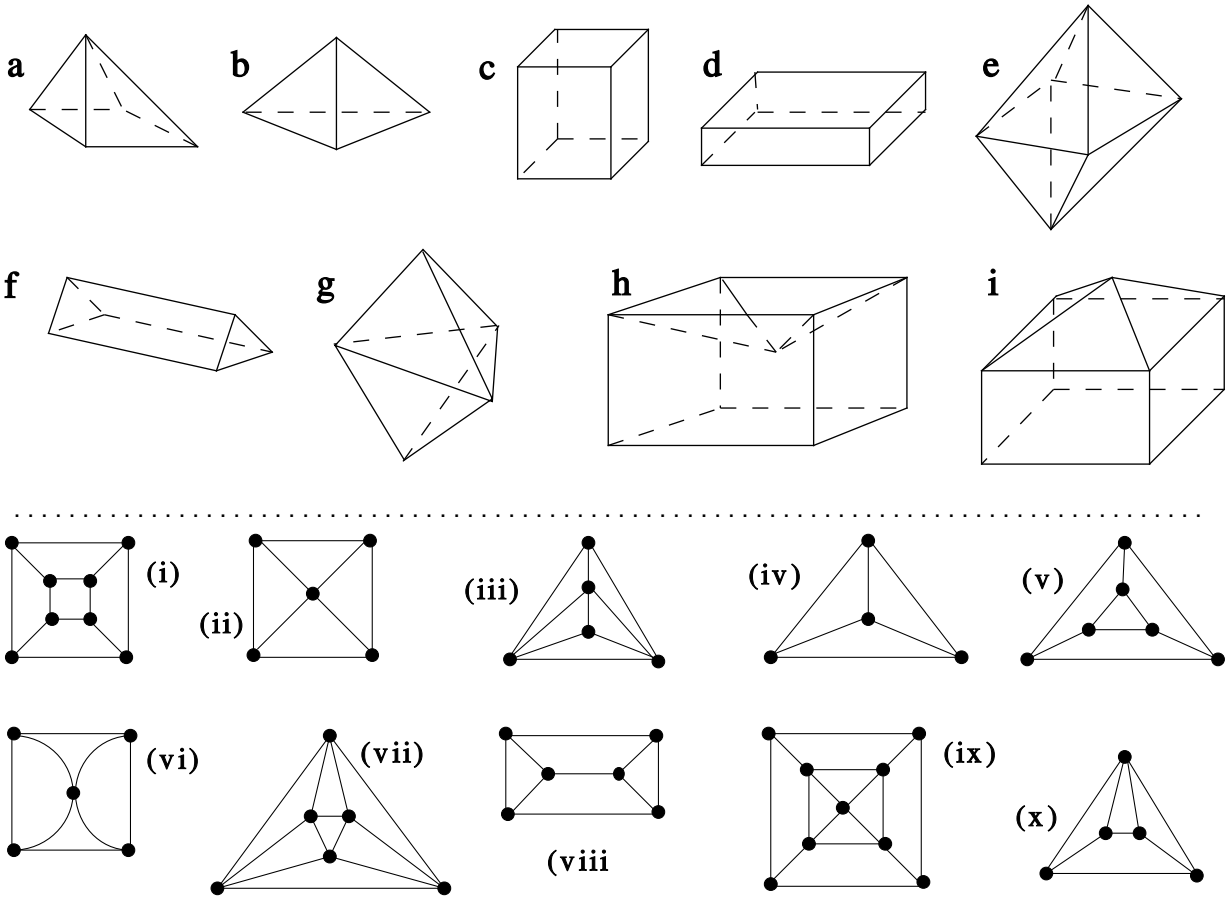
(a) 

(b) 

4. Which of the graphs 1a, 1d and 1e are planar? In each case you need to either draw an isomorphic plane graph or show the graph is non-planar.

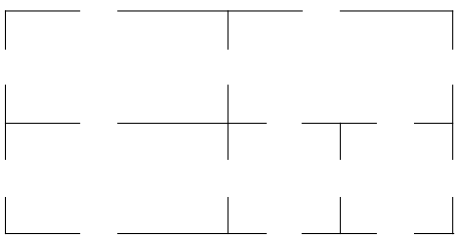
   Notice that the graphs in 1b, 1c and 1f are plane graphs, and hence are planar.

5. Match each of the following polyhedra, **a** to **i** with one or more of the appropriate plane graph(s), **(i)** to **(x)**.

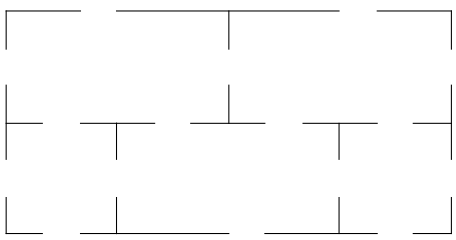


6. Use Euler's Theorem to decide which of the polyhedra it is possible to travel every edge exactly once, visit each vertex (corner), but not step off the edge onto the face.

7. Each of the figures below is the floor plan of a five-room house. House (a) has 15 walls while House (b) has 16 walls, and each wall contains a door. Is it possible to walk through every door of House (a) exactly once? Either show how to do it or **prove** that it is impossible. Repeat for House (b).



HOUSE (a)



HOUSE (b)