

2.

	No. of edges	No. of vertices	No. of odd vertices	No. of even vertices	Degree Sum $N_G$
1.	2	2	0	2	4
2.	1	2	2	0	2
3.	3	3	0	3	6
4.	10	10	6	4	20
5.	9	10	10	0	18
6.	6	5	0	5	12
7.	8	5	2	3	16
8.	6	7	6	1	12
9.	6	6	2	4	12
10.	7	5	2	3	14

3.  $N_G = 2E$ . In calculating the degree sum, we count each edge exactly twice, once for each of its two ends.

4. Conjecture: In any Graph, there is always an even number of vertices of odd degree.

Let  $N_{\text{odd}}$  be the sum of the degrees of the vertices of odd degree and let  $N_{\text{even}}$  be the sum of the degrees of the vertices of even degree. From Question 3, the degree sum  $N_G = d_1 + \dots + d_V$  is even. However, the sum  $N_{\text{even}}$  of the degrees of the vertices of even degree is even, so it follows from the equation  $N_{\text{odd}} + N_{\text{even}} = N_G$  that the sum  $N_{\text{odd}}$  of the degrees of the vertices of odd degree is also even. The only way the sum of a collection of odd numbers can be even is if there is an even number of them. Hence there is an even number of vertices of odd degree.

5.  $f(1) = F, f(2) = E, f(3) = A, f(4) = B, f(5) = C, f(6) = D.$   
 $g(1) = F, g(2) = E, g(3) = D, g(4) = C, g(5) = B, g(6) = A.$

6. Isomorphism 1  $4 \rightarrow D, 5 \rightarrow A, 1 \rightarrow B, 2 \rightarrow C, 3 \rightarrow E$   
 Isomorphism 2  $4 \rightarrow D, 5 \rightarrow A, 1 \rightarrow B, 2 \rightarrow E, 3 \rightarrow C$

7. (a) 
$$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(b) 
$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(c) If we list the vertices of Graph (D) in either of the orderings given by the two isomorphisms of Question 6, then we must get the same adjacency matrix. This is because an isomorphism preserves the number of edges joining corresponding vertices. So the possible orderings are:  $BCEDA$  and  $BECDA$

8. In (E) no pair of vertices of degree 2 are adjacent

In (F) there are two such pairs.

In (G) the two vertices of degree 3 are adjacent.

In (H) the two vertices of degree 3 are not adjacent.

Graph (I) has a circuit containing exactly 3 edges.

Graph (I) has a vertex of degree 2 adjacent to both vertices of degree 3.

Graph (K) has a circuit with 4 edges. Graph (K) has 2 adjacent vertices of degree 2 each of which is adjacent to a vertex of degree 3. (Others are possible.)

9. (a) Matrix for (M)

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Matrix for (N)

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

(b) Are the graphs are isomorphic? No. There are circuits with 3 edges in Graph (M) but there are no circuits with 3 edges in Graph (N).