The following second order recurrence relations have solutions as indicated:

(i) $T_n - 6T_{n-1} + 9T_{n-2} = 0 \quad n \geqslant 2 \quad T_0 = -1 \quad T_1 = -2$, has solution:

$$T_n = \tfrac{1}{3} \times n \times 3^n - 3^n$$

(ii) $U_n + U_{n-1} + \frac{1}{4}U_{n-2} = 0 \quad n \geqslant 2 \quad U_0 = 1 \quad U_1 = -1$, has solution:

$$U_n = \left(-\tfrac{1}{2}\right)^n + n \times \left(-\tfrac{1}{2}\right)^n$$

(iii) $F_n - F_{n-1} - F_{n-2} = 0 \qquad F_0 = 1 \quad F_1 = 1 \quad n \geqslant 2$, has solution:

$$F_n = \tfrac{1}{\sqrt{5}}\left(\tfrac{1+\sqrt{5}}{2}\right)^{n+1} + \tfrac{1}{\sqrt{5}}\left(\tfrac{1-\sqrt{5}}{2}\right)^{n+1}$$ (The famous Fibonacci sequence)

**1.** To which generic Big-$O$ classes do $T_n$, $U_n$ and $F_n$ belong?

**2.** (a) What is the range of "$\boldsymbol{c}$" values in the Big-O definition for $T_n$.
(b) What is the range of "$\boldsymbol{c}$" values in the Big-O definition for $U_n$.
(c) What is the range of "$\boldsymbol{c}$" values in the Big-O definition for $F_n$.

**3.** Show $2n^2 + 2n\log_2 n - 3n + 6 \in O(n^2)$.
Find the lowest half-integer value for "$c$".

**4.** Show $2n^2 + \frac{5n}{3} + 12 \in O(n^2)$.
Find the lowest quarter-integer value for "$c$".

**5.** Arrange the following Big-O classes in increasing order:
(Use set containment "$\subset$" to order the Big-O classes.)

$O(N)$ $O(N^2)$ $O(N^3)$ $O(2^N)$ $O(3^N)$ $O(N \times 2^N)$ $O(N^2 \times 2^N)$ $O(N^3 \times 2^N)$

Filling in the following table could help.

| $N$ | $N^2$ | $N^3$ | $2^N$ | $3^N$ | $N \times 2^N$ | $N^2 \times 2^N$ | $N^3 \times 2^N$ |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |
| 7 | | | | | | | |
| 8 | | | | | | | |
| 9 | | | | | | | |
| 10 | | | | | | | |
| 15 | | | | | | | |
| 20 | | | | | | | |
| 30 | | | | | | | |