1. Standard form : $S_n - 2S_{n-1} = 3n$

Homogeneous version : $S_n^H - 2S_{n-1}^H = 0$

Characteristic equation: $a - 2 = 0 \implies a - 2 = 0$ so $a = 2$

General homogeneous solution: $S_n^H = A \times 2^n$

Particular solution – try: $S_n^P = Bn + C$:

$$
\begin{aligned}
\text{LHS}= & S_n^P - 2S_{n-1}^P = 3n \\
\implies & Bn + C - 2(B \times (n-1) + C) = 3n \\
\implies & Bn + C - 2(Bn - B + C) = 3n \\
\implies & Bn + C - 2Bn + 2B - 2C = 3n \\
\implies & -Bn + 2B - C = 3n \\
\implies & -B = 3 \quad \& \quad 2B - C = 0 \quad i.e.\ B = -3 \ \&\ C = -6
\end{aligned}
$$

General solution is: $S_n = A \times 2^n - 3n - 6 \qquad (= S_n^H + S_n^P)$

Using the initial condition, $S_0 = 2$:

$$
\begin{aligned}
S_0 = 2 \implies & A - 6 = 2 \\
\implies & A = 8
\end{aligned}
$$

Solution: $\boxed{S_n = 8 \times 2^n - 3n - 6}$

CHECK: $S_0 = 8 - 0 - 6 = 2$ ✓

$$
\begin{aligned}
\text{Also} \quad S_n - 2S_{n-1} &= 8 \times 2^n - 3n - 6 - 2(8 \times 2^{n-1} - 3(n-1) - 6) \\
&= 8 \times 2^n - 3n - 6 - 2(8 \times 2^{n-1} - 3n + 3 - 6) \\
&= 8 \times 2^n - 2 \times 8 \times 2^{n-1} - 3n - 6 + 6n + 6 \\
&= 0 + 3n \\
&= 3n \quad \text{as required.}
\end{aligned}
$$

So, $S_n = 8 \times 2^n - 3n - 6$ is the answer to the initial value problem.

2. Standard form: $T_n - T_{n-1} = 3n$

Homogeneous version: $T_n^H - T_{n-1}^H = 0$

Characteristic equation: $a - 1 = 0 \implies a = 1$

General homogeneous solution: $T_n^H = A \times 1^n = A$

Particular solution – normally try – $T_n^P = Bn + C$ but because A and C are the same "type" of item we need to apply the **n-times rule**:

so we try $T_n^P = B \times n^2 + C \times n$

$$
\begin{aligned}
\text{LHS of the recurrence} &= T_n^P - T_{n-1}^P \\
&= Bn^2 + Cn - \left[B \times (n-1)^2 + C \times (n-1)\right] \\
&= Bn^2 + Cn - B \times (n^2 - 2n + 1) - Cn + C \\
&= Bn^2 + Cn - Bn^2 + 2Bn - B - Cn + C \\
&= 2Bn + -B + C \\
\text{RHS of the recurrence} &= 3n
\end{aligned}
$$

so $2B = 3$ and $-B + C = 0$ i.e. $B = \frac{3}{2}$ $C = \frac{3}{2}$

so $T_n^P = \frac{3}{2}n^2 + \frac{3}{2}n$

General solution is: $T_n = A + \frac{3}{2}n^2 + \frac{3}{2}n \quad (= T_n^H + T_n^P)$

Using the initial condition we have: $T_0 = 2 \implies A = 2$

Solution: $\boxed{T_n = 2 + \frac{3}{2}n^2 + \frac{3}{2}n}$ or $\boxed{T_n = \frac{3}{2}n^2 + \frac{3}{2}n + 2}$

CHECK: $T_0 = 2 + 0 = 2 \quad \checkmark$

$$
\begin{aligned}
\text{Also} \quad T_n - T_{n-1} &= \tfrac{3}{2}n^2 + \tfrac{3}{2}n + 2 - \left[\tfrac{3}{2}(n-1)^2 + \tfrac{3}{2}(n-1) + 2\right] \\
&= \tfrac{3}{2}n^2 + \tfrac{3}{2}n + 2 - \left[\tfrac{3}{2}(n^2 - 2n + 1) + \tfrac{3}{2}n - \tfrac{3}{2} + 2\right] \\
&= \tfrac{3}{2}n^2 + \tfrac{3}{2}n + 2 - \tfrac{3}{2}n^2 + 3n - \tfrac{3}{2} - \tfrac{3}{2}n + \tfrac{3}{2} - 2 \\
&= 3n \quad \checkmark
\end{aligned}
$$

So, $T_n = \frac{3}{2}n^2 + \frac{3}{2}n + 2$ is the answer to the initial value problem.

3. Homogeneous version: $I_n^H - I_{n-1}^H = 0$

Characteristic equation : $a - 1 = 0 \implies a = 1$

General homogeneous solution: $I_n^H = A$.

Particular solution – try: $I_n^P = Bn^2 + Cn$. [this is the n-times rule]

$Bn^2 + Cn - B \times (n-1)^2 - C \times (n-1) = \frac{n}{2} - \frac{1}{2}$.

$Bn^2 + Cn - B \times (n^2 - 2n + 1) - Cn + C = \frac{n}{2} - \frac{1}{2}$

$Bn^2 - Bn^2 + 2Bn - B + c = 2Bn - B + C = \frac{n}{2} - \frac{1}{2}$

So $2B = \frac{1}{2}$ and $-B + C = -\frac{1}{2} \implies B = \frac{1}{4}$ and $C = -\frac{1}{4}$.

Particular solution is: $I_n^P = \frac{1}{4}n^2 - \frac{1}{4}n$

General solution: $I_n = A + \frac{1}{4}n^2 - \frac{1}{4}n$

Using the initial condition $I_3 = \frac{3}{2}$ $\quad A + \frac{9}{4} - \frac{3}{4} = \frac{3}{2} \implies A + \frac{6}{4} = \frac{3}{2}$

So $A = 0$.

Solution: $\boxed{I_n = \frac{1}{4}n^2 - \frac{1}{4}n}$.

$$
\begin{aligned}
I_{100} &= \frac{1}{4} \times 100^2 - \frac{1}{4} \times 100 \\
&= \frac{10000}{4} - 25 = 2500 - 25 = 2475
\end{aligned}
$$