1. Rewrite each of the following expressions using Σ–notation

   For (d) and (e) use Σ–notation for both sides of the expression.

   (a) $1+2+3+\cdots+n$

   (b) $1^2+2^2+3^2+\cdots+n^2$

   (c) $1+3+5+\cdots+(2n-1)$

   (d) $(1^5+2^5+\cdots+n^5)+(1^7+2^7+\cdots+n^7)=2(1+2+\cdots+n)^4$

   (e) $1^3+2^3+3^3+\cdots+n^3=(1+2+3+\cdots+n)^2$

2. Re-write the following in Σ–notation and then use the Basic Rules to write them as separate summations.

   (a) $1\times 2+3\times 4+5\times 6+\cdots+(2n-1)\times 2n.$

   (b) $1^2+3^2+5^2+\cdots+(2n-1)^2.$

DEFINITION: $\mathbb{N}$ is the set of all natural numbers and $\mathbb{R}$ is the set of all real numbers.

For functions f and g, $f:\mathbb{N}\to\mathbb{R}$ and $g:\mathbb{N}\to\mathbb{R}$:

$f\in\mathbf{O}(g)$—if there are positive real numbers c and M such that:

$$|f(n)|\leqslant c|g(n)| \quad \text{for all } n\geq M.$$

We adopt a further shorthand:

if $g(n)=n$, for all $n\in\mathbb{N}$ then we write $\mathbf{O}(n)$ for $\mathbf{O}(g)$

if $g(n)=n^2$, for all $n\in\mathbb{N}$ then we write $\mathbf{O}(n^2)$ for $\mathbf{O}(g)$

3. Refer to Example 5* from Lecture 3 or using the definition above, write down the following Big-**O** definitions.

   (a) $2n^2+6\in\mathbf{O}(n^2)$

   (b) $n^2+2n\log n+\frac{1}{2}\in\mathbf{O}(n^3)$

   (c) $\frac{1}{2}n^3-6n^2+\frac{5}{3}\in\mathbf{O}(n^2)$

   [The statements in (a) & (b) are true while (c) is false.]

4. Prove that $2n^2+6\in\mathbf{O}(n^2)$

5. Prove that $3n^3-2n^2+\frac{1}{2}\in\mathbf{O}(n^3)$

6. Prove that $\frac{1}{3}n^2-2n\log n+\frac{1}{6}n-2\in\mathbf{O}(n^2)$

7. (a) Show that for any non-negative α,β,γ that $f\in\mathbf{O}(n^2)$ where $f(n)=\alpha n^2+\beta n+\gamma$.

   (b) What modification to your proof could you make to show $f\in\mathbf{O}(n^2)$ if we allow α,β or γ to be negative.