

This assignment is due on WEDNESDAY MARCH 28th at 5pm.

Put your assignment in one of the boxes labelled CSE2DMO, on Level 3 of PS2.

Please remember that your 3 Discrete Maths assignments are compulsory.

Statement of Originality. All assignments in this subject must begin with the following statement which must be signed and dated by you:

DECLARATION: *This assignment has not been copied at all or in part from other students' work.*

Name: _____

Signature: _____

Date: _____

1. (a) Write the following in Σ -notation:

(i) $1 + 2 + 4 + 8 + \dots + 2^{n-1} + 2^n$ (ii) $\frac{1}{10} + \frac{1}{15} + \frac{1}{20} + \dots + \frac{1}{95} + \frac{1}{100}$

- (b) Use iteration to calculate S_2 and S_3 given that

$$S_n = n \times S_{n-1} + 2^n \quad S_1 = 1$$

Show details of your calculations.

2. Use Induction to prove

$$2 + 4 + 6 + \dots + 2n = n \times (n + 1) \quad \text{for } n \geq 1$$

3. (a) Number the functions 1→5 from slowest to fastest growth.

Function	2^n	$n!$	3^n	n^3	$n^2 2^n$
Number					

- (b) Prove that: $2.2n^2 + 1.1n \log_2(n) - 20n + 15 \in O(n^2)$

- (c) To which generic Big O class does $\sum_{k=1}^{n-1} k$ belong?

4. Consider the multinomial coefficient: $\binom{10}{a, b, c, d} = \frac{10!}{a! \times b! \times c! \times d!}$

- (a) Referring to the expression above, complete the following:

(i) $a + b + c + d = \dots$ (ii) $\dots \leq a \leq \dots$

- (b) Write a simplified version (which involves no summation) of: $\sum_{r=1}^n \binom{n}{r} = \dots$

- (c) How many different arrangements of the characters GLENELG are there? Briefly explain your answer.

- (d) How many different 6-letter arrangements of the characters GLENELG are there? See Practice Class 1. Briefly explain your answer.

- (e) How many different 5-letter arrangements of the characters GLENELG are there? Show your working.

5. This question deals with strings using the characters a, b, c . Complete the following table.

	Length: $n = 3$	Length: $n = 8$	Length: n
How many strings of length ... ?			
How many contain no a 's?			
How many contain one a ?			
How many contain two a 's?			
How many contain three a 's?			
How many are palindromes?			

A palindrome is a string or word that reads backward the same as forward, eg ABBA and 10101 and 101101 are three palindromes.

For reasons of simplicity, the third column will use n to be greater than or equal to three.

6. (a) Find the solution to: $I_n - I_{n-1} = 0$ $I_1 = \frac{1}{2}$ for $n \geq 1$

(b) Solve and check: $J_n - J_{n-2} = 0$ $J_0 = 0$ $J_1 = \frac{1}{2}$ for $n \geq 2$

7. (a) Use SELECTION Sort to write down the lists for Pass 3, Pass 4 and Pass 5. Use a star to indicate the sorted elements of the list.

List to be sorted	After Pass 1	After Pass 2	After Pass 3	After Pass 4	After Pass 5
Hash	Bubble*	Bubble*			
Insert	Insert	Hash*			
Bubble	Hash	Insert			
Quick	Quick	Quick			
Select	Select	Select			
Merge	Merge	Merge			

(b) Referring to (a) above, complete the following table.

	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5
Number of Comparisons					
Number of Exchanges					

(c) For a general n -element unsorted list, how many passes are required to completely sort the list using Selection Sort?