1.

LIST 1	LIST 2	LIST 3	LIST 4
→Mike	→Geoff	Alan*	Alan*
Sid	Joan	Geoff*	Geoff*
Geoff	Harvey	→Joan	Harvey*
Joan	Alan	Harvey	Joan*
Peter	Mike*	Mike*	Mike*
Harvey	→Sid	Peter*	Peter*
Alan	Peter	Sid*	Sid*
	6 *comparisons*	4 *comparisons*	1 *comparison*

2.

LIST 1	LIST 2	LIST 3	LIST 4
→1	1*	1*	1*
2	→2	2*	2*
3	3	→3	3*
4	4	4	→4
⋮	⋮	⋮	⋮
n	n	n	n
	$n-1$ *comparisons*	$n-2$ *comparisons*	$n-3$ *comparisons*

3. **(a)** $(n-1)+(n-2)$

(b) $(n-1)+(n-2)+(n-3)$

(c) $(n-1)+(n-2)+(n-3)+\cdots+3+2+1$.

4. **(a)** (i) 1, 2 (ii) 2, 1.

(b) Each takes 1 comparison to sort. Thus $Q(2)=\frac{1+1}{2}=1$.

(c) (i) 1, 2, 3 (ii) 1, 3, 2 (iii) 2, 1, 3 (iv) 2, 3, 1 (v) 3, 1, 2 (vi) 3, 2, 1.

(d) For the two cases which start with 1, after the first pass *(2 comparisons)* we find that 1 stays at the start and we are left with an "unordered" list of 2 numbers, which takes a further comparison to sort. Thus each of these two cases take 3 comparisons. In a similar way, for the two cases starting with 3, 3 moves to the end and we also require 3 comparisons. In the two permutations which start with 2, after the first pass 2 is placed between the other two numbers and hence the list is sorted after this pass. Thus for these two cases there are only 2 comparisons.

It now follows that $Q(3)=\dfrac{(3\times 4)+(2\times 2)}{6}=\dfrac{8}{3}$.

5. $Q(2) = -8 + 6(1 + \frac{1}{2}) = -8 + \frac{6\times 3}{2} = -8 + 9 = 1.$
$Q(3) = -12 + 8(1 + \frac{1}{2} + \frac{1}{3}) = -12 + \frac{8\times 11}{6} = -12 + \frac{44}{3} = \frac{8}{3}.$

6. (a) $\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$

(b) $\displaystyle\sum_{r=2}^{n} \frac{1}{r}$

(c) The total area of the shaded rectangles is less than the area under the curve,

so $\displaystyle\sum_{r=2}^{n} \frac{1}{r} \leq \log_e(2)\log(n).$

(d)
$$\begin{aligned}|Q(n)| &= \left|-4n + (2n+2)\sum_{r=1}^{n}\frac{1}{r}\right| \\ &\leq 4n + (2n+2)\sum_{r=1}^{n}\frac{1}{r} \qquad \text{(by the } \triangle \text{ inequality)} \\ &\leq 4n + (2n+2)(1+\log_e(2)\log(n)) \qquad \text{(by (c))} \\ &\leq 4n\log(n) + 4n(1+\log_e(2)\log(n)), \qquad \text{for } n \geq 2 \\ &\leq 4n\log(n) + 4n\log(n) + 4\log_e(2)n\log(n), \quad \text{for } n \geq 2 \\ &= 4(2+\log_e(2))n\log(n).\end{aligned}$$

Hence $Q(n) \in \boldsymbol{O}(n\log(n))$. Note that many other solutions, including many giving better values of c, can be found. The best value of c is $2\log_e(2)$.