

1.	<u>LIST 5</u>		<u>List 6</u>		<u>List 7</u>
	ACT		ACT		ACT
	NSW		NSW		NSW
	QLD		QLD		NT
	$\xrightarrow{1 \text{ comp}}$ TAS	$\xrightarrow{3 \text{ comps}}$	SA	$\xrightarrow{5 \text{ comps}}$	QLD
	<u>VIC</u>		TAS		SA
	SA		<u>VIC</u>		TAS
	NT		NT		<u>VIC</u>
	WA		WA		WA

2.

<u>LIST 1</u>		<u>LIST 2</u>		<u>LIST 3</u>		<u>LIST 4</u>	\vdots		<u>LIST $n-1$</u>		<u>LIST n</u>
<u>n</u>		$n-1$		$n-2$		$n-3$	\vdots		2		1
$n-1$	$\xrightarrow{1 \text{ comp}}$	<u>n</u>	$\xrightarrow{2 \text{ comps}}$	$n-1$	$\xrightarrow{3 \text{ comps}}$	$n-2$	\vdots	$\xrightarrow{n-2}$	3	$\xrightarrow{n-1}$	2
$n-2$		$n-2$		<u>n</u>		$n-1$	\vdots	comps	4	comps	3
$n-3$		$n-3$		$n-3$		<u>n</u>	\vdots		5		4
$n-4$		$n-4$		$n-4$		$n-4$	\vdots		\vdots		\vdots
\vdots		\vdots		\vdots		\vdots	\vdots		<u>n</u>		$n-1$
1		1		1		1	\vdots		1		<u>n</u>

3. (a) $I(4) = 1 + 2 + 3$. (b) $I(5) = 1 + 2 + 3 + 4$.

4. (a) As 1 more element is sorted at each pass, the process will terminate with LIST n . Since for each $i \in \{1, 2, \dots, n\}$ LIST i adds $i - 1$ comparisons we have

$$I(n) = 1 + 2 + 3 + \dots + (n - 1)$$

(b) $2I(n) = (n - 1)n$, so $I(n) = \frac{n(n-1)}{2} = \frac{1}{2}n^2 - \frac{1}{2}n$.

5. $I(n) \in \mathcal{O}(n^2)$ if there exist positive numbers c and M such that

$$|I(n)| \leq cn^2 \text{ for } n \geq M.$$

6. Note that for $n \geq 1$, $\frac{n(n-1)}{2} \geq 0$ so that $|I(n)| = I(n) = \frac{1}{2}n^2 - \frac{1}{2}n \leq \frac{1}{2}n^2$.

Choosing $c = \frac{1}{2}$ and $M = 1$, we have; $|I(n)| \leq cn^2$ for $n \geq M$,

so, $I(n) \in \mathcal{O}(n^2)$.

(Using the triangle inequality could result in values of $c = 1$ and $M = 1$.)

7. (a)

List 1		List 2		List 3
<u>1</u>	$\xrightarrow{1 \text{ comp}}$	1	$\xrightarrow{1 \text{ comp}}$	1
2		<u>2</u>		2
3		3		<u>3</u>

(b) $J(3) = 1 + 1 = 2$

(c) $J(4) = 1 + 1 + 1 = 3$

(d) $J(n) = n - 1$. Hence, for any $n \in \mathbb{N}$, $|J(n)| = J(n) = n - 1 \leq n$ so (using $c = 1$ and $M = 1$) $J(n) \in \mathcal{O}(n)$.

8. (a) 123, 132, 213, 231, 312, 321.

(b)

LIST 1	LIST 2	LIST 3	LIST 1	LIST 2	LIST 3
<u>1</u>	1	1	<u>2</u>	1	1
2	<u>2</u>	2	1	<u>2</u>	2
3	3	<u>3</u>	3	3	<u>3</u>

(c) 2 extra comparisons (one for each list)

(d)

LIST 1	LIST 2	LIST 3	LIST 1	LIST 2	LIST 3
<u>1</u>	1	1	<u>3</u>	1	1
3	<u>3</u>	2	1	<u>3</u>	2
2	2	<u>3</u>	2	2	<u>3</u>

This requires $4 = 2 \times 2$ extra comparisons (2 for each list)

(e)

LIST 1	LIST 2	LIST 3	LIST 1	LIST 2	LIST 3
<u>2</u>	2	1	<u>3</u>	2	1
3	<u>3</u>	2	2	<u>3</u>	2
1	1	<u>3</u>	1	1	<u>3</u>

This requires $4 = 2 \times 2$ extra comparisons (2 for each list).

(f) The lists ending with 3 require a total of $T_2 + 2$, those ending in 2 require a total of $T_2 + 4$ and those ending in 1 require a total of $T_2 + 4$, giving altogether $3T_2 + 2 + 4 + 4 = 3T_2 + 2(1 + 2 + 2)$.

9. We have to consider $n - 1$ different insertions, one to move from List 1 to List 2, one to move from List 2 to List 3 and so on. The fewest comparisons at each stage is 1, so the minimum total number is $(n - 1) \times 1 = J(n)$. In moving from List i to List $i + 1$ the $(i + 1)$ st element must be compared with some of the first i elements. The most comparisons required is i . Thus $I(n) = 1 + 2 + \dots + (n - 1)$ gives the maximum number of comparisons needed.