1. (a) Write the following in Σ-notation.

   i. $x+\frac{x^3}{3}+\frac{x^5}{5}+\cdots+\frac{x^{2n-1}}{2n-1}$

   ii. $\frac{5}{3}+\frac{7}{4}+\frac{9}{5}+\cdots+\frac{99}{50}$

   (b) Use iteration to calculate $S(2), S(3)$ & $S(4)$ given

   $$S(n)=\frac{2}{(n-1)^2}\times S(n-1) \qquad S(1)=2$$

2. Use Induction to prove:

   $1\times 1!+2\times 2!+3\times 3!+\cdots+n\times n!=(n+1)!-1$ for each $n\in\mathbb{N}$

3. (a) Number the functions below, 1→5 from slowest to fastest growth.

Function	$\log(n)$	$n\log(n)$	n^2	$n^{1.5}$	$n^{\frac{1}{3}}$
Number					

   (b) Prove that $3n^2\log_2 n+\frac{4}{3}n\log_2 n-10\log_2 n\in O(n^2\log_2 n)$, with the choice for $c=3\frac{1}{2}$.

4. Solve and check the following recurrence relation:

   $S_n-5S_{n-1}+4S_{n-2}=0 \quad S_0=1 \quad S_1=3$

5. Find the solution to the following recurrence relation:

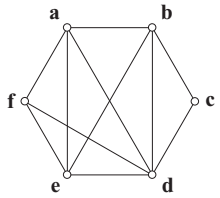
   $$I_n-I_{n-1}=\tfrac{n}{2}-\tfrac{1}{2} \qquad I_3=\tfrac{3}{2}$$

   [For a particular solution you will need to try: $I_n^P=B\times n^2+C\times n$]

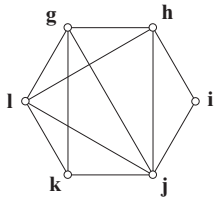
   No need to do a check, but check your answer gives: $I_{100}=2475$

6. (a) Write down an isomorphism from **G_1** to **G_2**.

   $f(\mathbf{a})=\ldots\ f(\mathbf{b})=\ldots\ f(\mathbf{c})=\ldots\ f(\mathbf{d})=\ldots\ f(\mathbf{e})=\ldots\ f(\mathbf{f})=\ldots$



G_1



G_2

   (b) Use Fleury's algorithm to write down an Eulerian path for **G_1**

   To avoid ambiguity, use the following rules:

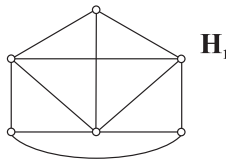
   From any possible vertices to start, choose the one whose label occurs first in the alphabet.

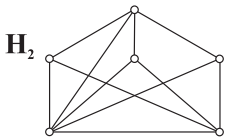
   If there is a choice of edges at some step, then choose the one that travels to the vertex with the alphabetically earliest label.

   Only cross a bridge if there is no alternative.

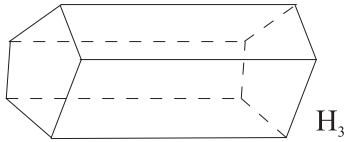
   Write down each vertex as it is visited.

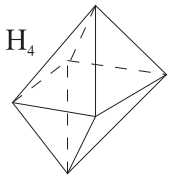
7. (a) Give one reason why **H_1** and **H_2** are **not** isomorphic.


H_1


H_2

   (b) Draw a plane graph representation for the two polyhedra pictured below


H_3



8. (a) Use Selection Sort to write down the lists for Pass 2, Pass 3, Pass 4 and Pass 5. Use *'s to indicate sorted elements.

List to be sorted	After Pass 1	After Pass 2	After Pass 3	After Pass 4	After Pass 5
Don	Amazon*				
Tigris	Tigris				
Yellow	Yellow				
Nile	Nile				
Yarra	Yarra				
Amazon	Don				

   (b) How many comparisons are required in total for the above sorting process, including the first pass?

9. (a) Use Insertion Sort to write down the lists for Pass 2, Pass 3, Pass 4 and Pass 5. Use a line to separate sorted elements from the unsorted elements.

List to be sorted	After Pass 1	After Pass 2	After Pass 3	After Pass 4	After Pass 5
Don	Don				
Tigris	Tigris				
Yellow	Yellow				
Nile	Nile				
Yarra	Yarra				
Amazon	Amazon				

(b) How many comparisons are required in total for the above sorting process, including the first pass?

10. Use Kruskal's Algorithm to find a minimal spanning tree for the graph with:

vertices v_1 v_2 v_3 v_4 v_5 v_6

edges v_1v_2 v_1v_4 v_1v_6 v_2v_3 v_2v_6 v_3v_4 v_3v_5 v_4v_5 v_5v_6

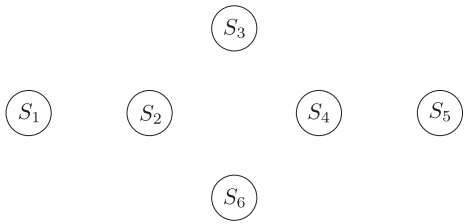
weights 3 7 8 10 6 4 2 5 9

11. Consider the finite state machine given by the transition table:

state table	0	1
S_1	S_6	S_2
S_2	S_3	S_6
S_3	S_6	S_4
S_4	S_5	S_6
S_5	S_6	S_3
S_6	S_6	S_6

initial state: S_1

acceptor state: S_3



(a) Draw the state diagram for this machine using the layout above.

(b) Describe the language accepted by this machine.

(c) Construct a finite state machine which accepts only the following words: $11, 10, 101$

12. (a) How many different arrangements of the characters INHIBITION are there? Briefly explain your answer.

(b) What is the coefficient of $x^2y^3z^4$ in the expansion of $(x+3y-2z)^9$

(c) This question deals with bit strings, that is strings consisting of a's and b's. Complete the following table.

	Length: $\mathbf{n=3}$	Length: $\mathbf{n=8}$	Length: $\mathbf{n}$
How many bit strings of length ...?			
How many contain no a's?			
How many contain one a?			
How many contain three b's?			
How many are palindromes?			

A palindrome is a string or word that reads backward the same as forward,

eg ABBA and 10101 and 101101 are three palindromes.

For reasons of simplicity, the third column will use $\mathbf{n}$ to be greater than or equal to three.

13. For the Finite State Machine of this question, the input language consists of strings of 0's and 1's. The language to be accepted consists of strings which end in (at least) two zeros.

(a) Write the language to be accepted as a set.

(b) Create a Finite State Machine which accepts this language.

14. This question deals with binomial coefficients.

Prove that: $\sum_{j=0}^{j=2n} \binom{2n}{j} = 2^{2n}$

Allocation of Marks

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Marks	7	6	7	6	6	4	5	8	8	6	6	10	4	5