1. (a) i. $\displaystyle\sum_{k=1}^{n} \frac{x^{2k-1}}{2k-1}$ ii. $\displaystyle\sum_{k=3}^{50} \frac{2k-1}{k}$

   (b) $S(2) = \frac{2}{1^2}\times S(1) = 2\times 2 = 4$

   $S(3) = \frac{2}{2^2}\times S(2) = \frac{2}{4}\times 4 = 2$ $S(4) = \frac{2}{3^2}\times S(3) = \frac{2}{9}\times 2 = \frac{4}{9}$

2. Let Claim(n) be: $1\times 1! + 2\times 2! + 3\times 3! + \cdots + n\times n! = (n+1)! - 1$

   For $n = 1$: LHS $= 1\times 1! = 1$

   RHS $= 2! - 1 = 2 - 1 = 1$ ✓

   For $n = k$: Assume $1\times 1! + 2\times 2! + 3\times 3! + \cdots + k\times k! = (k+1)! - 1$ (is true)

   For $n = k+1$:

$$
\begin{aligned}
\text{LHS} &= 1\times 1! + 2\times 2! + 3\times 3! + \cdots + k\times k! + (k+1)\times(k+1)! \\
&= \big[1\times 1! + 2\times 2! + 3\times 3! + \cdots + k\times k!\big] + (k+1)\times(k+1)! \\
&= (k+1)! - 1 + (k+1)\times(k+1)! \qquad \text{by the inductive assumption} \\
&= (k+1)![1 + k + 1] - 1 \\
&= (k+1)!(k+2) - 1 \\
&= (k+2)! - 1 \quad \text{as required.}
\end{aligned}
$$

   By Induction: $1\times 1! + 2\times 2! + 3\times 3! + \cdots + n\times n! = (n+1)! - 1$

3. (a)

Function	$\log(n)$	$n\log(n)$	n^2	$n^{1.5}$	$n^{\frac{1}{3}}$
Number	1	3	5	4	2

   (b)

$$
\begin{aligned}
&\left|3n^2\log_2(n) + \tfrac{4}{3}n\log_2(n) - 10\log_2(n)\right| \\
&\leqslant \left|3n^2\log_2(n)\right| + \left|\tfrac{4}{3}n\log_2(n)\right| + \left|-10\log_2(n)\right| \qquad \text{by the } \triangle \text{ inequality} \\
&= 3n^2\log_2(n) + \tfrac{4}{3}n\log_2(n) + 10\log_2(n) \\
&\leqslant 3n^2\log_2(n) + \tfrac{1}{4}n^2\log_2(n) + \tfrac{1}{4}n^2\log_2(n) \\
&= 3\tfrac{1}{2}n^2\log_2(n)
\end{aligned}
$$

   provided $\frac{4}{3}n\log_2(n) \leqslant \frac{1}{4}n^2\log_2(n)$ & $10\log_2(n) \leqslant \frac{1}{4}n^2\log_2 n$

   i.e. $\frac{16}{3}n\log_2(n) \leqslant n^2\log_2(n)$ & $40\log_2(n) \leqslant n^2\log_2(n)$

   i.e. $\frac{16}{3} \leqslant n$ & $40 \leqslant n^2$

   i.e. $\frac{16}{3} \leqslant n$ & $\sqrt{40} \leqslant n$.

   Choosing $c = 3\frac{1}{2}$ $M = \max\{\frac{16}{3}, \sqrt{40}\}$,

   $3n^2\log_2(n) + \frac{4}{3}n\log_2(n) - 10\log_2(n) \in O(n^2\log_2(n))$

4. For the recurrence $S_n - 5S_{n-1} + 4S_{n-2} = 0$

   Characteristic equation: $a^2 - 5a + 4 = 0$

   $\Longrightarrow (a-4)(a-1) = 0 \Longrightarrow a = 4$, or $a = 1$

   General solution: $S_n = A(4)^n + B(1)^n = A4^n + B$

   Using initial conditions: $S_0 = A(4)^0 + B = A + B = 1$

   $S_1 = A(4)^1 + B \Longrightarrow 4A + B = 3$

   Solving these gives: $A = \dfrac{2}{3},\ B = \dfrac{1}{3}$

   Solution is: $\boxed{S_n = \dfrac{2}{3}(4)^n + \dfrac{1}{3}}$

   CHECK: $S_0 = \frac{2}{3}(4)^0 + \frac{1}{3} = \frac{2}{3} + \frac{1}{3} = 1$

   $S_1 = \frac{2}{3}(4)^1 + \frac{1}{3} = \frac{4}{3} + \frac{1}{3} = \frac{9}{3} = 3$ ✓

   LHS: $S_n - 5S_{n-1} + 4S_{n-2}$

$$
\begin{aligned}
&= [\tfrac{2}{3}(4)^n + \tfrac{1}{3}] - 5[\tfrac{2}{3}(4)^{n-1} + \tfrac{1}{3}] + 4[\tfrac{2}{3}(4)^{n-2} + \tfrac{1}{3}] \\
&= [\tfrac{2}{3}(4)^n] - 5[\tfrac{2}{3}(4)^{n-1}] + 4[\tfrac{2}{3}(4)^{n-2}] + [\tfrac{1}{3} - \tfrac{5}{3} + \tfrac{4}{3}] \\
&= (4)^{n-2}[\tfrac{2}{3}\times 4^2 - \tfrac{10}{3}\times 4^1 + \tfrac{8}{3}] + \mathbf{0} \\
&= (4)^{n-2}[\tfrac{32}{3} - \tfrac{40}{3} + \tfrac{8}{3}] \\
&= (4)^{n-2}[0] = 0 \quad \text{as required.}
\end{aligned}
$$

5. Homogeneous version: $I_n^H - I_{n-1}^H = 0$

Characteristic equation: $a - 1 = 0 \implies a = 1$

General homogeneous solution: $I_n^H = A$.

Particular solution, try: $I_n^P = B\times n^2 + C\times n$. [this is the n-times rule]

$B\times n^2 + C\times n - B(n-1)^2 - C(n-1) = \frac{n}{2} - \frac{1}{2}$.

$B\times n^2 + C\times n - B(n^2 - 2n + 1) - C\times n + C = \frac{n}{2} - \frac{1}{2}$

$B\times n^2 - B\times n^2 + 2B\times n - B + C = 2B\times n - B + C = \frac{n}{2} - \frac{1}{2}$

So: $2B = \frac{1}{2}$ and $-B + C = -\frac{1}{2} \implies B = \frac{1}{4}$ and $C = -\frac{1}{4}$.

Particular solution: $I_n^P = \frac{1}{4}n^2 - \frac{1}{4}n$

General solution: $I_n = A + \frac{1}{4}n^2 - \frac{1}{4}n$

Using the initial condition: $I_3 = \frac{3}{2}$ $\quad A + \frac{9}{4} - \frac{3}{4} = \frac{3}{2}$

$\implies A + \frac{6}{4} = \frac{3}{2}$ So: $A = 0$

Solution: $\boxed{I_n = \frac{1}{4}n^2 - \frac{1}{4}n}$

$I_{100} = \frac{1}{4}\times 100^2 - \frac{1}{4}\times 100 = \frac{10000}{4} - 25 = 2500 - 25 = 2475$

6. (a) $f(\mathbf{a}) = \mathbf{g}$ $\quad f(\mathbf{b}) = \mathbf{h}$ $\quad f(\mathbf{c}) = \mathbf{i}$ $\quad f(\mathbf{d}) = \mathbf{j}$ $\quad f(\mathbf{e}) = \mathbf{l}$ $\quad f(\mathbf{f}) = \mathbf{k}$

OR $f(\mathbf{a}) = \mathbf{l}$ $\quad f(\mathbf{b}) = \mathbf{h}$ $\quad f(\mathbf{c}) = \mathbf{i}$ $\quad f(\mathbf{d}) = \mathbf{j}$ $\quad f(\mathbf{e}) = \mathbf{g}$ $\quad f(\mathbf{f}) = \mathbf{k}$.

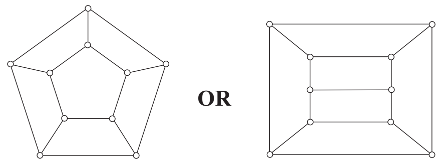
(b) **d a b c d b e a f d e f**

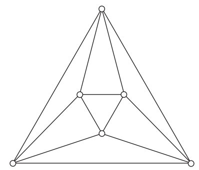
7. (a) Suitable reasons:

(1) In H_1 the vertices of degree 4 are adjacent.

(2) In H_1 two vertices of degree 3 are adjacent.

(b) Plane graph representation for H_3 — Plane graph for H_4





8. (a)

List to be sorted	After Pass 1	After Pass 2	After Pass 3	After Pass 4	After Pass 5
Don	Amazon*	Amazon*	Amazon*	Amazon*	Amazon*
Tigris	Tigris	Don*	Don*	Don*	Don*
Yellow	Yellow	Yellow	Nile*	Nile*	Nile*
Nile	Nile	Nile	Yellow	Tigris*	Tigris*
Yarra	Yarra	Yarra	Yarra	Yarra	Yarra*
Amazon	Don	Tigris	Tigris	Yellow	Yellow

(b) Total of 15 comparisons.

9. (a)

List to be sorted	After Pass 1	After Pass 2	After Pass 3	After Pass 4	After Pass 5
Don	Don	Don	Don	Don	Amazon
Tigris	Tigris	Tigris	Nile	Nile	Don
Yellow	Yellow	Yellow	Tigris	Tigris	Nile
Nile	Nile	Nile	Yellow	Yarra	Tigris
Yarra	Yarra	Yarra	Yarra	Yellow	Yarra
Amazon	Amazon	Amazon	Amazon	Amazon	Yellow

(b) Total of 12 comparisons.

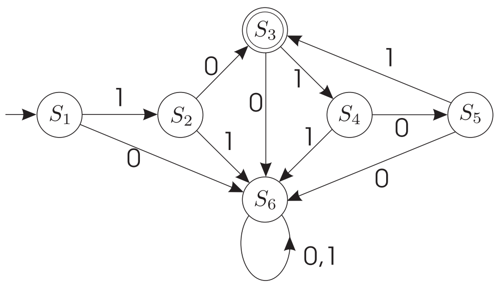
10. $e_1 = v_3v_5$ $e_2 = v_1v_2$ $e_3 = v_3v_4$ $e_4 = v_4v_5$

$e_5 = v_2v_6$ $e_6 = v_1v_4$ $e_7 = v_1v_6$ $e_8 = v_5v_6$ $e_9 = v_2v_3$

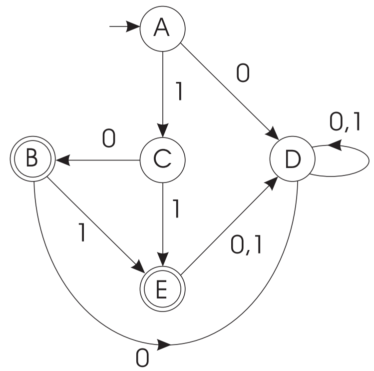
	N(1)	N(2)	N(3)	N(4)	N(5)	N(6)	E	W
init.	1	2	3	4	5	6	$\emptyset$	0
1.	1	2	3	4	3	6	e_1	2
2.	1	1	3	4	3	6	e_1, e_2	5
3.	1	1	3	3	3	6	e_1, e_2, e_3	9
4.	1	1	3	3	3	6	e_1, e_2, e_3	9
5.	1	1	3	3	3	1	e_1, e_2, e_3, e_5	15
6.	1	1	1	1	1	1	e_1, e_2, e_3, e_5, e_6	22

Minimal spanning tree has edges: e_1, e_2, e_3, e_5, e_6; Weight: 22.

11. (a)



(c)



(b) The language accepted is: $10(101)^n$ for $n \geq 0$.

12. (a) ANSWER: $\binom{10}{4,2,1,1,1,1} = \dfrac{10!}{4!\times 2!\times 1!\times 1!\times 1!} = 75600$

REASON: There are 10 characters: 4 I's, 2 N's, 1 H, 1 B, 1 T, 1 O.

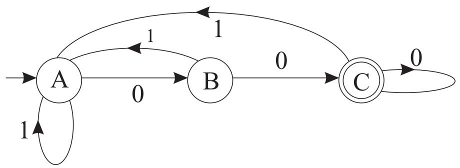
(b) $\binom{9}{2,3,4} 1^2 3^3 (-2)^4 = 1260 * 27 * 16 = 544320$

(c)

	Length $\mathbf{n=3}$	Length $\mathbf{n=8}$	Length $\mathbf{n}$
Bit strings of length	$\mathbf{2^3}$	$\mathbf{2^8}$	$\mathbf{2^n}$
Contain no a's	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Contain one a	$\mathbf{3 = {}^3C_1}$	$\mathbf{8 = {}^8C_1}$	$\mathbf{n = {}^nC_1}$
Contain three b's	$\mathbf{1 = {}^3C_3}$	$\mathbf{{}^8C_3}$	$\mathbf{{}^nC_3}$
Palindromes	$\mathbf{4}$	$\mathbf{16 = 2^4}$	$\mathbf{2^{(n/2)}}$, $\mathbf{n}$ even. $\mathbf{2^{(n+1)/2}}$, $\mathbf{n}$ odd.

13. (a) $\{w00:$ where w is any word$\}$ OR (0 | 1)*00

(b)



14. Prove that: $\displaystyle\sum_{j=1}^{j=2n} \binom{2n}{j} = 2^{2n}$

Using the Binomial Theorem: $(A+B)^{2n} = \displaystyle\sum_{j=0}^{2n} \binom{2n}{j} A^{2n-j} B^j$

Now set $A = 1$ and $B = 1$, to get: $2^{2n} = \displaystyle\sum_{j=0}^{2n} \binom{2n}{j}$

Good luck with Discrete Mathematics and all your other subjects.
Please inform me of any errors you find in these solutions.

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