

COMBINATORICS

1. Complete the following table.

	Length: $n = 3$	Length: $n = 8$	Length: n
How many bit strings of length ... ?			
How many contain no 1's ?			
How many contain one 1 ?			
How many contain two 1's ?			
How many contain three 1's ?			
How many are palindromes ...			Case 1: when n is odd ? Case 2: when n is even ?

A palindrome is a string or word that reads backward the same as forward, eg ABBA and 10101 and 101101 are three palindromes.

For reasons of simplicity, the third column will use n to be greater than or equal to three.

2. When used in connection with counting, what is the difference between *permutation* and *combination*?
3. (a) How many functions from $\{1, 2, 3, 4\}$ to $\{a, b, c, d\}$ are there? Think combinatorially.
 (b) How many one-to-one functions from $\{1, 2, 3, 4\}$ to $\{a, b, c, d\}$ are there?
 (**Note:** A function is **one-to-one** if $f(x) \neq f(y)$ whenever $x \neq y$.)
 (c) How many functions from $\{1, 2, 3\}$ to $\{a, b, c, d, e\}$ are there? Think combinatorially.
 (d) How many one-to-one functions from $\{1, 2, 3\}$ to $\{a, b, c, d, e\}$ are there?
4. Ordinary registration plates in Victoria contain 3 capital letters followed by 3 digits.
 - (a) How many different registration plates are there? (Assuming that all letters and digits may be used.)
 - (b) How many would there be if letters could not be repeated but digits could be?
 - (c) How many would there be if neither letters nor digits could be repeated?
5. (a) How many different registration plates containing 1, 2 or 3 letters followed by 4 digits are there? Indicate clearly where you use the Addition and Multiplication Rules.
 (b) How many different registration plates containing 1, 2 or 3 letters followed by 1, 2, 3 or 4 digits are there? Indicate clearly where you use the Addition and Multiplication Rules.
6. (a) Expand $(2x - y)^4$ using the Binomial Theorem.
 (b) What is the coefficient of $x^2y^4z^3$ in the expansion of $(x + 2y - 3z)^9$?
7. In this question you will count how many ways can we choose 9 people out of 13 and then distribute them into 3 teams, the first having 2 members, the second having 3 members and the third having 4 members.
 - (a) Solve this problem using a step-by-step binomial approach.
 - (b) Solve this problem using a multinomial approach.

8. (a) Write down all the permutations of x, y, z .
Write them down as strings, and try to write them down in a logical order.
- (b) Write down all the 2-permutations of x, y, z , again try to write them down in a systematic order.
9. (a) Give a formula for $\binom{n}{r_1, r_2, \dots, r_k}$ in terms of factorials.
- (b) What condition on r_1, r_2, \dots, r_k must hold for $\binom{n}{r_1, r_2, \dots, r_k}$ to be defined?
10. In this question, calculate the answers, don't leave your answers in terms of factorials.
- (a) How many different arrangements of GLENELG are there?
(Hint: Each arrangement is an 7-letter word. Think in terms of labelling 7 positions with the labels G, L, E and N .)
- (b) How many 6 letter words can be formed from the letters of GLENELG. (Indicate where you use the Addition Rule.)
- (c) What do you notice about your answers to (a) and (b). Is this a fluke?
11. (a) In the game of bridge there are four hands of 13 cards dealt from a deck of 52 cards. How many different bridge deals are possible?
- (b) In the game of rummy each deal consists of four hands of 9 cards dealt from a deck of 52 cards. How many different rummy deals are possible?
12. In the following question, indicate clearly each time you use the Addition or Multiplication Rule.
Consider a normal pack of 52 cards.
- (a) How many 5-card hands are there?
Of these, how many consist
- (b) only of hearts?
- (c) only of a single suit?
- (d) of 2 clubs and 3 hearts?
- (e) of 2 of one suit and 3 of another suit?
- (f) of 2 clubs, 2 hearts and 1 diamond?
- (g) of 2 of one suit, 2 of another suit and 1 of a third suit?
- (h) of 2 aces and 3 kings?
- (i) of 2 of one denomination and 3 of another denomination? (e.g. 2 "tens" and 3 "Queens".)
13. How many 5-card hands from a normal pack of 52 cards contain
- (a) exactly one pair?
- (b) no pairs?
14. Consider "words" of any 6 letters. How many contain at least one vowel?
Hint: First count how many do **not** contain a vowel. Indicate where you use the Addition Rule.
15. Suppose we draw a card from a pack of 52 cards and replace it before we draw the next one. In how many ways can 10 such draws be made (taking note of the order) so that the 10th card drawn is the same as one of the previous nine? Indicate clearly where you use the Addition and Multiplication Rules.
Hint: Count how many sequences of 10 draws there are in which the 10th card is not a repetition. To do this, **first** choose the 10th card.