1.

n	sum	total	**n**	sum	total
1	1	1	1	1^2	1
2	$1+3$	$\mathbf{4}$	2	1^2+2^2	$\mathbf{5}$
3	$1+3+5$	$\mathbf{9}=3^2$	3	$1^2+2^2+3^2$	$\mathbf{14}$
4	$1+3+5+7$	$\mathbf{16}=4^2$	4	$1^2+2^2+3^2+4^2$	$\mathbf{30}$
5	$1+3+5+7+9$	$\mathbf{25}=5^2$	5	$1^2+2^2+3^2+4^2+5^2$	$\mathbf{55}$
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
k	$1+3+5+\cdots+(\mathbf{2k-1})$	$\mathbf{k^2}$	k	$1^2+2^2+3^2+\cdots+<\ldots\ldots>$	$\mathbf{\frac{1}{6}k(k+1)(2k+1)}$

2. (a) $A(2) = -\frac{1}{3}A(1) = -\frac{2}{3}$

$A(3) = -\frac{1}{3}A(2) = \frac{2}{9}$

$A(4) = -\frac{1}{3}A(3) = -\frac{2}{27}$

(b) $S(2) = S(1) + \frac{1}{2}S(0) = 2\frac{1}{2}$

$S(3) = 2\frac{1}{2} + \frac{1}{2}\times 2 = 3\frac{1}{2}$

$S(4) = 3\frac{1}{2} + \frac{1}{2}\times 2\frac{1}{2} = 4\frac{3}{4}$

(c) $T(2) = 1\frac{1}{2}$ $T(3) = 2$ $T(4) = 2\frac{3}{4}$

(d) $K_4 = K_3 - K_2 - K_0 = 0$ $K_5 = K_4 - K_3 - K_1 = -4$ $K_6 = K_5 - K_4 - K_2 = -6$

3. Let Claim(n) be " $1+3+5+\cdots+(2n-1) = n^2$ " $n \geqslant 1$

For n=1: LHS is 1 RHS is $1^2 = 1$ ✓

For n=k: Assume $1+3+5+\cdots+(2k-1) = k^2$ (is true)

We need to show that for **n=k+1**, claim $(k+1)$ is true.

i.e. $1+3+5+\cdots+(2k-1)+(2k+1) = (k+1)^2$

For n=k+1:

$$\begin{aligned} LHS &= 1+3+5+\cdots+(2k-1)+(2k+1) \\ &= \left[1+3+5+\cdots+(2k-1)\right]+(2k+1) \\ &= k^2+(2k+1) \qquad \text{by the Inductive Assumption} \\ &= k^2+2k+1 \\ &= (k+1)^2 \end{aligned}$$

Hence by Induction: $1+3+5+\cdots+(2n-1) = n^2$

4.
$$\begin{aligned} f(4,2) &= 2f(3,2) - f(3,1) \\ &= 2\left[2f(2,2) - f(2,1)\right] - \left[2f(2,1) - f(2,0)\right] \\ &= 4f(2,2) - 4f(2,1) + f(2,0) \\ &= 4\cdot 2 - 4f(2,1) + 1 \\ &= 9 - 4f(2,1) \\ &= 9 - 4\left[2f(1,1) - f(1,0)\right] \\ &= 9 - 8f(1,1) + 4f(1,0) \\ &= 9 - 16 + 4 = -\mathbf{3} \end{aligned}$$

5. Let Claim(n) be " $1^2+2^2+3^2+\cdots+n^2=\frac{1}{6}n(n+1)(2n+1)$ "

For n=1: LHS is $1^2=1$

RHS is $\frac{1}{6}\times1(1+1)(2\times1+1)=\frac{1}{6}\times2\times3=\frac{6}{6}=1$ ✓

For n=k:

Assume $1^2+2^2+3^2+\cdots+k^2=\frac{1}{6}\times k(k+1)(2k+1)$ (is true)

We need to show that for **n=k+1**, claim $(k+1)$ is true.

i.e. $1^2+2^2+3^2+\cdots+k^2+(k+1)^2=\frac{1}{6}\times(k+1)(k+2)(2k+3)$

For n=k+1: LHS = $1^2+2^2+3^2+\cdots+k^2+(k+1)^2$

$= \left[1^2+2^2+3^2+\cdots+k^2\right]+(k+1)^2$

$=\frac{1}{6}\times k(k+1)(2k+1)+(k+1)^2$ by the Inductive Assumption

For n=k+1: $= [k+1]\left(\frac{1}{6}\times k(2k+1)+(k+1)\right)$

$=(k+1)\frac{1}{6}\left(2k^2+k+6k+6\right)$

$=\frac{1}{6}(k+1)\left(2k^2+7k+6\right)$

$=\frac{1}{6}(k+1)(2k+3)(k+2)$

Hence by Induction: $1^2+2^2+3^2+\cdots+n^2=\frac{1}{6}n(n+1)(2n+1)$

6. (a) $n=1:$ $f(4)=\dfrac{(-1)^2+f(2)f(3)}{f(1)}$

$$=\frac{1+1\times2}{1}=3$$

$n=2:$ $f(5)=\dfrac{(-1)^3+f(3)f(4)}{f(2)}=\dfrac{-1+2\times3}{1}=5$

$n=3:$ $f(6)=\dfrac{(-1)^4+f(4)f(5)}{f(3)}=\dfrac{1+3\times5}{2}=8$

(b) $n=3:$ $f(6)=\dfrac{(-1)^4+f(4)f(5)}{f(3)}$

$$=\frac{1+\frac{(-1)^2+f(2)f(3)}{f(1)}\times\frac{(-1)^3+f(3)f(4)}{f(2)}}{2}$$

$$=\frac{1+\frac{1+2}{1}\times\frac{-1+2f(4)}{1}}{2}$$

$$=\frac{1+3\times\left[-1+2\frac{(-1)^2+f(2)f(3)}{f(1)}\right]}{2}$$

$$=\frac{1+3\times\left[-1+2\times3\right]}{2}$$

$$=8$$

(c) Obvious? No.

7. No solution given.