1. Complete the following table.

   There is something to fill in wherever you see the symbol $< \ldots\ldots >$

   The symbol $< ???>$ means think hard or move on.

n	sum	total	**n**	sum	total
1	1	1	1	1^2	1
2	$1+3$	$< \ldots\ldots >$	2	1^2+2^2	$< \ldots\ldots >$
3	$1+3+5$	$< \ldots\ldots >$	3	$1^2+2^2+3^2$	$< \ldots\ldots >$
4	$1+3+5+7$	$< \ldots\ldots >$	4	$1^2+2^2+3^2+4^2$	$< \ldots\ldots >$
5	$1+3+5+7+9$	$< \ldots\ldots >$	5	$1^2+2^2+3^2+4^2+5^2$	$< \ldots\ldots >$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	$1+3+5+\cdots+< \ldots\ldots >$	$< \ldots\ldots >$	k	$1^2+2^2+3^2+\cdots+< \ldots\ldots >$	$< ???>$

2. Use the following recurrence relations to calculate the next 3 terms.

   (a) $A(n) = -\frac{1}{3}A(n-1) \quad A(1) = 2$

   (b) $S(n) = S(n-1) + \frac{1}{2}S(n-2) \quad S(0) = 1 \quad S(1) = 2 \quad n \geqslant 2$

   (c) $T(n) = T(n-1) + \frac{1}{2}T(n-2) \quad T(0) = 1 \quad T(1) = 1 \quad n \geqslant 2$

   (d) $K_{n+4} = K_{n+3} - K_{n+2} - K_n \quad K_0 = 1 \quad K_1 = 1 \quad K_2 = 2 \quad K_3 = 3 \qquad n \geqslant 0$

3. Use Induction to prove $\quad 1+3+5+\cdots+(2n-1) = n^2 \qquad$ for $n \geqslant 1$

4. The function f is defined recursively for all non-negative integers k and positive integers n, with $n \geqslant k$, as follows:

$$\begin{cases} f(n,k) = 2f(n-1,k) - f(n-1,k-1) \\ f(n,n) = 2 \ \text{ and } \ f(n,0) = 1 \end{cases}$$

   Calculate $f(4,2)$

5. Use Induction to prove $1^2+2^2+3^2+\ldots+n^2 = \frac{1}{6}n(n+1)(2n+1) \qquad$ for $n \geqslant 1$

6. The function f is defined for all positive integers n as follows:

$$f(n+3) = \frac{(-1)^{n+1} + f(n+1)\ f(n+2)}{f(n)} \qquad f(1) = f(2) = 1 \quad \text{and} \quad f(3) = 2$$

   (a) Use iteration to evaluate $f(6)$.

   (b) Use recursion to evaluate $f(6)$.

   (c) Is it obvious to you that $f(n)$ is an integer for each positive integer n?

7. The terms of the Fibonacci sequence are given by:

$$f(n+2) = f(n+1) + f(n) \qquad f(1) = f(2) = 1, \text{ where } n \text{ is a positive integer.}$$

   Using Induction, prove that:

$$\begin{cases} f(n+3) = \dfrac{(-1)^{n+1} + f(n+1)\ f(n+2)}{f(n)} \\ f(1) = f(2) = 1 \quad \text{and} \quad f(3) = 2 \end{cases}$$

   So f in Question 5 defines the Fibonacci sequence.