1.

| | **n = 3** | **n = 8** | **n** |
|---|---|---|---|
| How many bit strings of length ...? | $2^3 = \mathbf{8}$ | $2^8 = \mathbf{256}$ | $\mathbf{2^n}$ |
| How many contain no 1's? | ${}^3C_0 = \mathbf{1}$ | ${}^8C_0 = \mathbf{1}$ | ${}^nC_0 = \mathbf{1}$ |
| How many contain one 1? | ${}^3C_1 = \mathbf{3}$ | ${}^8C_1 = \mathbf{8}$ | ${}^nC_1 = \mathbf{n}$ |
| How many contain two 1's? | ${}^3C_2 = \mathbf{3}$ | ${}^8C_2 = \mathbf{28}$ | ${}^nC_2 = \frac{\mathbf{n \times (n-1)}}{\mathbf{2}}$ |
| How many contain three 1's? | ${}^3C_3 = \mathbf{1}$ | ${}^8C_3 = \mathbf{56}$ | ${}^nC_3 = \frac{\mathbf{n \times (n-1)(n-2)}}{\mathbf{6}}$ |
| How many are palindromes? | **4** | $\mathbf{2^4}$ | $\mathbf{2^{(n+1)/2}}$ for **n** odd. $\mathbf{2^{n/2}}$ for **n** even. |

2. For "permutation", order is important. For "combination", order is not relevant.

3. (a) We can map 1 to any one of the four points in $\{a,b,c,d\}$, and similarly for 2, 3 and 4. Thus, by the Multiplication Rule, there is a total of $4 \times 4 \times 4 \times 4 = 4^4 = 256$ functions.

(b) We can map 1 to any one of 4 points in $\{a,b,c,d\}$. Then we can map 2 to any of the remaining 3, and then 3 to any of the remaining 2 points in $\{a,b,c,d\}$. Thus, by Multiplication Rule, there is a total of $4 \times 3 \times 2 \times 1 = 4! = 24$.

(c) We can map 1 to any one of 5 points in $\{a,b,c,d,e\}$. Then we can map 2 to any of 5 and then 3 to any of 5 points. Thus, there is a total of $5 \times 5 \times 5 = 125$.

(d) We can map 1 to any one of 5 points in $\{a,b,c,d,e\}$. Then we can map 2 to any of remaining 4 and then 3 to any of remaining 3 points. Thus, there is a total of $5 \times 4 \times 3 = 60$.

4. (a) **Letter part:** $26 \times 26 \times 26$. **Digit part:** $10 \times 10 \times 10$.
**Total:** $26 \times 26 \times 26 \times 10 \times 10 \times 10$ (Multiplication Rule).

(b) **Letter part:** $26 \times 25 \times 24$. **Digit Part:** $10^3$. **Total:** $26 \times 25 \times 24 \times 10^3$.

(c) $26 \times 25 \times 24 \times 10 \times 9 \times 8$.

5. (a) **Letter part.** 1 letter: 26, 2 letters: $26^2$, 3 letters: $26^3$. Since these three sets are disjoint, the total is $26 + 26^2 + 26^3$, by the Addition Rule. Thus, since there are $10^4$ digit parts, we have $(26 + 26^2 + 26^3) \times 10^4$ choices in all, by the Multiplication Rule.

(b) By the Addition Rule, the number of different digit parts is $(10 + 10^2 + 10^3 + 10^4)$. Thus, by the Multiplication Rule, the total number is $(26 + 26^2 + 26^3) \times (10 + 10^2 + 10^3 + 10^4)$.

6. (a) $\binom{4}{0}(2x)^4(-y)^0 + \binom{4}{1}(2x)^3(-y)^1 + \binom{4}{2}(2x)^2(-y)^2 + \binom{4}{3}(2x)^1(-y)^3 + \binom{4}{4}(2x)^0(-y)^4$
$= 16x^4 - 32x^3y + 24x^2y^2 - 8xy^3 + y^4$

(b) $-\binom{9}{2,4,3} \times 16 \times 27$ OR $\binom{9}{2,4,3} \times 2^4 \times (-3)^3$ OR $\frac{9!}{2!4!3!} \times 2^4 \times (-3)^3$

7. (a) We can choose the 9 people in $\binom{13}{9}$ ways. Then, having chosen the 9, we can choose the first team in $\binom{9}{2}$ ways. The second team can then be chosen in $\binom{7}{3}$ ways from the remaining 7. This leaves us with no further choice—the remaining 4 make up the third ream. By the Multiplication Rule, the total number of choices is

$$\binom{13}{9}\binom{9}{2}\binom{7}{3} = \frac{13!}{9!4!} \times \frac{9!}{2!7!} \times \frac{7!}{3!4!}$$

(b) From the 13 people, select 4 <u>not</u> to chosen, then choose 2 for the first team, 3 for the next team and 4 the remaining team. This can be done in:

$$\binom{13}{4,2,3,4} \text{ ways. Which equals } \frac{13!}{4! \times 2! \times 3! \times 4!} = 900900$$

8. (a) $xyz \quad xzy \quad yxz \quad yzx \quad zxy \quad zyx$

   (b) $xy \quad xz \quad yx \quad yz \quad zx \quad zy$

9. (a) $\binom{n}{r_1,r_2,\ldots,r_k} = \frac{n!}{r_1!\times r_2!\times r_3!\times\cdots\times r_k!}$

   (b) For $\binom{n}{r_1,r_2,\ldots,r_k}$ to be defined: $r_1+r_2+r_3+\cdots+r_k = \boldsymbol{n}$

10. (a) **630**. There are $\binom{7}{2,2,2,1} = \frac{7!}{2!2!2!1!} = \frac{7\times6\times5\times4\times3\times2}{2\times2\times2} = 630$ arrangements.

    (b) **630**. If we omit "G" there are $\frac{6!}{1!2!2!1!} = 180$ words, ie different arrangements of LENELG
    If we omit "L" there are $\frac{6!}{2!2!1!1!} = 180$ words, ie different arrangements of GENELG
    If we omit "E" there are $\frac{6!}{1!2!2!1!} = 180$ words, ie different arrangements of GLNELG
    If we omit "N" there are $\frac{6!}{2!2!2!} = 90$ words, ie different arrangements of GLEELG
    Since these sets are pairwise disjoint we have (by the Addition Rule) a total of 630.

    (c) It is easy to create a 1-1 correspndence between the 7 letter strings and the 6 letter strings.

11. (a) Multinomial approach: $\binom{52}{13,13,13,13} = \frac{52!}{13!\times13!\times13!\times13!}$ is the quick and easy answer.
    Step-by-step binomial: $\binom{52}{13}\times\binom{39}{13}\times\binom{26}{13} = \frac{52!}{39!\times13!}\times\frac{39!}{26!\times13!}\times\frac{26!}{13!\times13!} = \frac{52!}{13!\times13!\times13!\times13!}$

    (b) Multinomial approach: $\binom{52}{9,9,9,9,16} = \frac{52!}{9!\times9!\times9!\times9!\times16!}$
    Step-by-step binomial: $\binom{52}{9}\times\binom{43}{9}\times\binom{34}{9}\times\binom{25}{9} = \frac{52!}{43!\times9!}\times\frac{43!}{34!\times9!}\times\frac{34!}{25!\times9!}\times\frac{25!}{16!\times9!} = \frac{52!}{9!\times9!\times9!\times9!\times16!}$

Brief answers only for the remaining questions, if you have doubts about these solutions, consult the lecturer.

12. (a) $C(52,5) = \frac{52!}{5!\times47!}$. (Order does not matter.)

    (b) We must choose from only the 13 hearts, hence there are $C(13,5)\frac{13!}{5!\times8!}$ hands.

    (c) $4\times C(13,5)$

    (d) $\binom{13}{2}\binom{13}{3}$

    (e) $4\times3\times\binom{13}{2}\binom{13}{3}$

    (f) $\binom{13}{2}\times\binom{13}{2}\times 13$ hands.

    (g) $\binom{4}{2}\times2\times\binom{13}{2}\times\binom{13}{2}\times13$

    (h) $\binom{4}{2}\binom{4}{3}(=24)$

    (i) $13\times12\times24$

13. (a) $13\times\binom{12}{3}\times\binom{4}{2}\times4^3$

    (b) $\binom{13}{5}\times4^5$

14. $26^6 - 21^6$

15. $52^{10} - 52\times51^9$