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La Trobe University Semester One Examination 2005

Student ID: Seat Number:

Unit Code: CSE21DMO Unit Name: Discrete Maths. and Computer Org.
 Paper No: 1 Paper Name: Discrete Mathematics
 Reading Time: 15 minutes Writing Time: 1 hour 30 minutes
 Examination Date: 28/06/2005 Examination Start Time: 14:00

No. of pages (including cover sheet): 14

ALLOWABLE MATERIALS
AND
INSTRUCTIONS TO CANDIDATES

- 1) This examination consists of 13 questions
13 typed pages and a blank page
- 2) Please check all pages are present
- 3) If you need more space than provided, use the reverse side of a page or use the extra page attached to the end of this booklet
- 4) Do not remove any pages
- 5) Mathematics department approved calculator
- 6) **Total Marks: 80**

Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
TOTAL	

This paper MUST NOT BE REMOVED from the examination venue

Question 1.

(4 + 3 = 7 marks)

(a) Write the following in Σ -notation:

(i) $21 + 26 + 31 + \dots + 91 + 96 + 101 = \dots\dots\dots$

(ii) $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \dots + \frac{1}{2n-4} + \frac{1}{2n-2} = \dots\dots\dots$

(b) Use iteration to calculate S_3, S_4 and S_5 given that

$$S_n = 4 \times S_{n-1} + \frac{1}{n} \times S_{n-2} \quad S_1 = 6, S_2 = 12$$

Show details of your calculations.

Question 2.

(6 marks)

Use Induction to prove

$$1 + 3 + 5 + \cdots + (2n - 3) + (2n - 1) = n^2 \quad \text{for } n \geq 1$$

Question 3.

(2 + 5 = 7 marks)

- (a) Number the functions 1→5 from slowest to fastest growth.

Function	$n^3 2^n$	$n!$	2^n	n^n	3^n
Number					

- (b) Prove that: $1.7n^2 + 2n \log_2(n) + 5n + 25 \in O(n^2)$
with the choice $c = 2$

[You may find Rule 1 or Rule 2 useful.]

Rule 1	If $d \geq 2$ then $n \log_2 n \geq d$ for $n \geq d$
Rule 2	If $d \geq 4$ then $n \geq d \log_2 n$ for $n \geq d^2$

Question 4.

(5 + 1 = 6 marks)

(a) Find the solution to: $I_n - I_{n-1} = \frac{1}{2}n + 2$ $I_1 = 3$ for $n \geq 2$

For the particular solution I_n^P you will need to try:

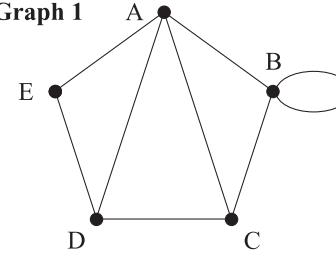
$$I_n^P = B \times n^2 + C \times n$$

No check is required.

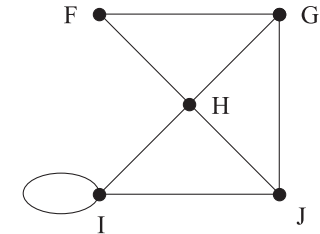
Question 5.

(2 + 2 = 4 marks)

Graph 1



Graph 2



(a) Write down an isomorphism from Graph 1 to Graph 2.

$f(\mathbf{A}) = \dots\dots$ $f(\mathbf{B}) = \dots\dots$ $f(\mathbf{C}) = \dots\dots$ $f(\mathbf{D}) = \dots\dots$ $f(\mathbf{E}) = \dots\dots$

(b)

Recall that Fleury's algorithm can be described as follows.

- (a) Start travelling along any edge, unless the graph has two vertices of odd degree, in which case start at an edge which begins at one of them.
- (b) Rub out the last edge and then travel over any edge which is not a bridge in the new graph (unless we are at a vertex of degree 1).

Use Fleury's algorithm to construct an Eulerian path in Graph 3.

Fleury's Algorithm, simplified version To avoid ambiguity, use the following rules:

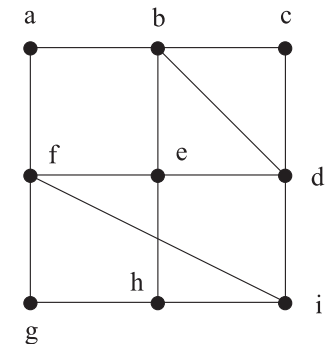
From any possible vertices to start, choose the one whose label occurs first in the alphabet.

If there is a choice of edges at some step, then choose the one that travels to the vertex with the alphabetically earliest label.

Only cross a bridge if there is no alternative.

Write down each vertex as it is visited.

VERTEX LIST FOR THE EULERIAN PATH:



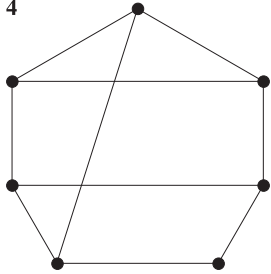
Graph 3

(b) Calculate the value of your answer at $n = 40$. (It should be 490.5)
Show details of your calculation.

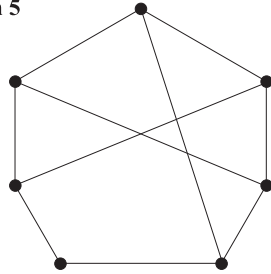
Question 6.

(2 + 3 = 5 marks)

Graph 4



Graph 5

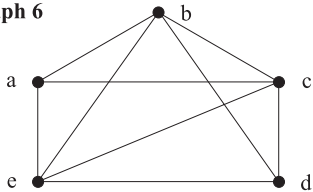


(a) Give one reason why Graph 4 and Graph 5 are **not** isomorphic.

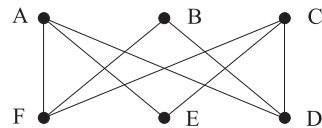
ANSWER:

(b) Draw an isomorphic plane graph for both Graph 6 and Graph 7.

Graph 6



Graph 7



(i) PLANE GRAPH isomorphic to **Graph 6**

(ii) PLANE GRAPH isomorphic to **Graph 7**

Question 7.

(4 + 4 + 1 = 9 marks)

(a) Use SELECTION Sort to write down the lists for Pass 2, Pass 3, Pass 4 and Pass 5. Use *'s to indicate sorted elements.

List to be sorted	After Pass 1	After Pass 2	After Pass 3	After Pass 4	After Pass 5
Loop	Arc*				
Set	Set				
Arc	Loop				
Tree	Tree				
Path	Path				
Edge	Edge				

Comparison table for the first pass.

Compare	Loop	Loop	Arc	Arc	Arc
With	Set	Arc	Tree	Path	Edge
Temp.	Loop	Arc	Arc	Arc	Arc

(b) Complete the comparison tables for Pass 2, Pass 3, Pass 4 and Pass 5.

Pass 2

Compare				
With				
Temp.				

Pass 3

Compare			
With			
Temp.			

Pass 4

Compare		
With		
Temp.		

Pass 5

Compare	
With	
Temp.	

(c) How many comparisons are required in total for the above sorting process?

Answer:

Question 8.

(6 marks)

Use Kruskal's Algorithm to find a minimal spanning tree for the graph with:

Vertices: $v_1 v_2 v_3 v_4 v_5 v_6$

Edges: $v_1v_3 v_1v_5 v_2v_5 v_2v_6 v_3v_4 v_3v_5 v_3v_6 v_4v_5 v_5v_6$

Weights: 9 6 2 1 3 5 7 4 8

(The edges and weights are listed in corresponding order.)

$e_1 = \dots\dots e_2 = \dots\dots e_3 = \dots\dots e_4 = \dots\dots e_5 = \dots\dots$

$e_6 = \dots\dots e_7 = \dots\dots e_8 = \dots\dots e_9 = \dots\dots$

	N(1)	N(2)	N(3)	N(4)	N(5)	N(6)	Edges	Weight
Init.	1	2	3	4	5	6	\emptyset	0

ANSWER:

The edge set for the minimal spanning tree consists of edges:

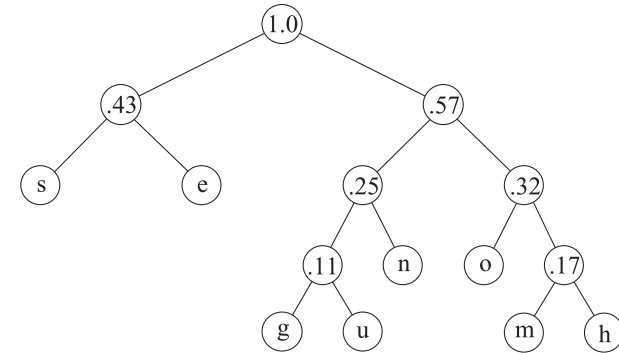
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The weight of the minimal spanning tree is:

Question 9.

(3 + 1 + 1 = 5 marks)

(a) Label the branches of the tree below to create a Huffman code and fill in the table below.



CHARACTER	CODE	CHARACTER	CODE
e		n	
g		o	
h		s	
m		u	

(b) Encode

e n o u g h

(c) Decode 1111110111011010000110101110100100

.....

Question 10.

(2 + 3 + 2 = 7 marks)

Consider the finite state machine given by the transition table below.

transition function	0	1
S_1	S_2	S_4
S_2	S_3	S_5
S_3	S_3	S_3
S_4	S_2	S_5
S_5	S_2	S_3

initial state: S_1
acceptor state: S_5

(a) (i) Complete the state diagram below.



(ii) Describe the language accepted by this machine.

LANGUAGE ACCEPTED IS:

(b) Construct a finite state machine which accepts only the following two words: 000, 1100.

Question 11.

(2 + 2 + 6 = 10 marks)

(a) How many different arrangements of the characters AUTOMATON are there? Briefly explain your answer.

ANSWER AND EXPLANATION:

(b) What is the coefficient of $x^3y^3z^6$ in the expansion of $(3x - y + 2z)^{12}$?

ANSWER:

(c) This question deals with bit strings, that is, strings of a's and b's. Complete the following table.

	Length: $n = 2$	Length: $n = 7$	Length: n
How many bit strings of length ... ?			
How many contain no b's?			
How many contain exactly two b's?			
How many contain exactly three b's?			
How many contain at least one b?			
How many are palindromes?			

A palindrome is a string or word that reads backward the same as forward, eg aaaa and acaca and abccba are three palindromes.

For reasons of simplicity, the third column will use n to be greater than or equal to three.

Question 12.

(4 marks)

This question deals with Finite State Machines.

INPUT LANGUAGE: Strings of 0's and 1's

LANGUAGE TO BE ACCEPTED: Strings which end with exactly two 0's.

The language to be accepted written in symbolic notation is: $((0|1)^*1)^*00$

Create a finite state machine which accepts the above language.

ANSWER:

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Question 13.

(4 marks)

This question deals with sequences and recurrence relations.

Work out a recurrence relation and an initial condition which describes the sequence given in the following table. Your answer should not include any factorials.

Sequence	S_1	S_2	S_3	S_4	S_n
Values	2	6	20	70	$\frac{(2n)!}{n! \times n!}$
	2C_1	4C_2	6C_3	8C_4	${}^{2n}C_n$