Unit Code: CSE21DMO **Paper Name: Discrete Mathematics** **Year: 2004**

Question 1. (4 + 3 = 7 marks)

(a) Write the following in Σ-notation:

(i) $16 + 18 + 20 + \cdots + 60 + 62 + 64 =$ .....

(ii) $\frac{1}{2} + \frac{1}{5} + \frac{1}{8} + \cdots + \frac{1}{3n-4} + \frac{1}{3n-1} =$ .....

(b) Use iteration to calculate S_3, S_4 and S_5 given that

$$S_n = 2 \times S_{n-1} \; + \frac{n}{n-1} \times S_{n-2} \qquad S_1 = 2, \;\; S_2 = 3$$

Show details of your calculations.

Question 2. (6 marks)

Use Induction to prove

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots+\frac{1}{n\times(n+1)}=\frac{n}{n+1} \quad \text{for} \quad n\geq 1$$

Question 3. (2 + 5 = 7 marks)

(a) Number the functions 1→5 from slowest to fastest growth.

Function	$\sqrt{n}$	n	$\log_2(n)$	$\log_2(n!)$	$n^{\frac{1}{3}}$
Number					

(b) Prove that: $.6n^2 \log_2(n) + \frac{1}{2}n^2 + 12n \log_2(n) + n + 5 \log_2(n) \in O\big(n^2 \log_2(n)\big)$ with the choice $c = 1$

[You may find Rule 1 or Rule 2 useful.**]**

Rule 1	If $d \geqslant 2$ then $n \log_2 n \geqslant d$	for $n \geqslant d$
Rule 2	If $d \geqslant 4$ then $n \geqslant d \log_2 n$	for $n \geqslant d^2$

Question 4. (5 + 1 = 6 marks)

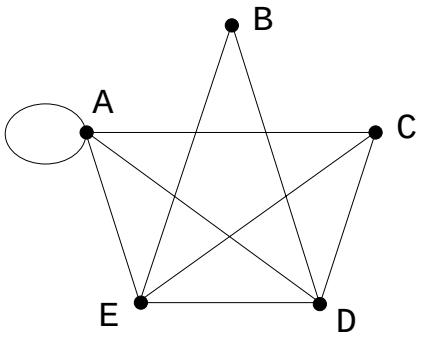
(a) Find the solution to: $I_n - I_{n-1} = 2n + 1 \qquad I_1 = 1 \qquad$ for $n \geqslant 2$

For the particular solution I_n^P you will need to try:

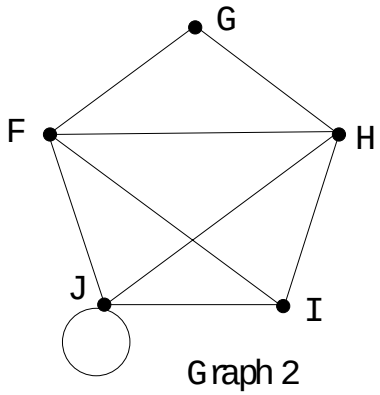
$I_n^P = B \times n^2 + C \times n$ **No check is required.**

(b) Calculate the value of your answer at $n = 30$. Show details of your calculation. (It should be 958)

Question 5. (2 + 2 = 4 marks)



Graph 1



Graph 2

(a) Write down an isomorphism from Graph 1 to Graph 2.

$f(\mathbf{A}) = \ldots\ldots \quad f(\mathbf{B}) = \ldots\ldots \quad f(\mathbf{C}) = \ldots\ldots \quad f(\mathbf{D}) = \ldots\ldots \quad f(\mathbf{E}) = \ldots\ldots$

(b)

Recall that Fleury's algorithm can be described as follows.

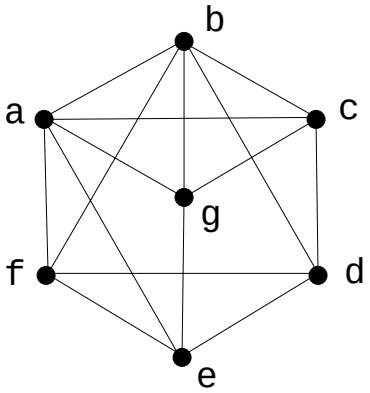
1. Start travelling along any edge, unless the graph has two vertices of odd degree, in which case start at an edge which begins at one of them.

2. Rub out the last edge and then travel over any edge which is not a bridge in the new graph (unless we are at a vertex of degree 1).

Use Fleury's algorithm to construct an Eulerian path in Graph 3. List the edges in the order in which you select them. At each stage where there is any choice, choose the edge which is earliest in the list below. (N.B. The edge "**xy**" is regarded as the same as the edge "**yx**" when being considered for selection.)

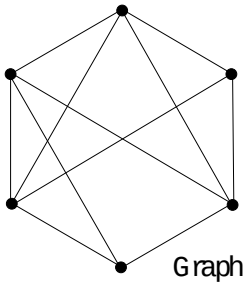
ab, ac, ae, af, ag, bc, bd, bf, bg, cd, cg, de, df, ef, eg

EDGE LIST FOR THE EULERIAN PATH:

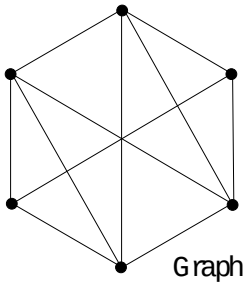


Graph 3

Question 6. (2 + 3 = 5 marks)



Graph 4

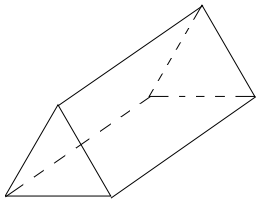


Graph 5

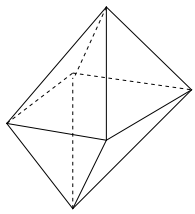
(a) Give one reason why Graph 4 and Graph 5 are **not** isomorphic.

ANSWER:

(b) Draw a plane graph representation for the two polyhedra depicted below.



A



B

(i) PLANE GRAPH FOR **A**

(ii) PLANE GRAPH FOR **B**

Question 7. (4 + 5 = 9 marks)

(a) Use INSERTION Sort to write down the lists for Pass 2, Pass 3, Pass 4 and Pass 5. Underline the last element of the sorted portion of the list.

List to be sorted	After Pass 1	After Pass 2	After Pass 3	After Pass 4	After Pass 5
Map	Ker				
Ker	Map				
Basis	Basis				
Vector	Vector				
Linear	Linear				
Dim	Dim				

(b) Referring to (a) above, complete the following table.

	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5
Number of Comparisons					
Number of Exchangess					

Question 8. (6 marks)

Use Kruskal's Algorithm to find a minimal spanning tree for the graph with:

Vertices: v_1 v_2 v_3 v_4 v_5 v_6

Edges: v_1v_2 v_1v_4 v_1v_6 v_2v_3 v_2v_4 v_2v_5 v_2v_6 v_3v_4 v_3v_5 v_5v_6

Weights: 2 9 5 7 4 6 1 10 3 8
(The edges and weights are listed in corresponding order.)

$e_1 =$ ..... $e_2 =$ ..... $e_3 =$ ..... $e_4 =$ ..... $e_5 =$ .....

$e_6 =$ ..... $e_7 =$ ..... $e_8 =$ ..... $e_9 =$ ..... $e_{10} =$ .....

	$N(1)$	$N(2)$	$N(3)$	$N(4)$	$N(5)$	$N(6)$	Edges	Weight
Initially	1	2	3	4	5	6	$\varnothing$	0

ANSWER:

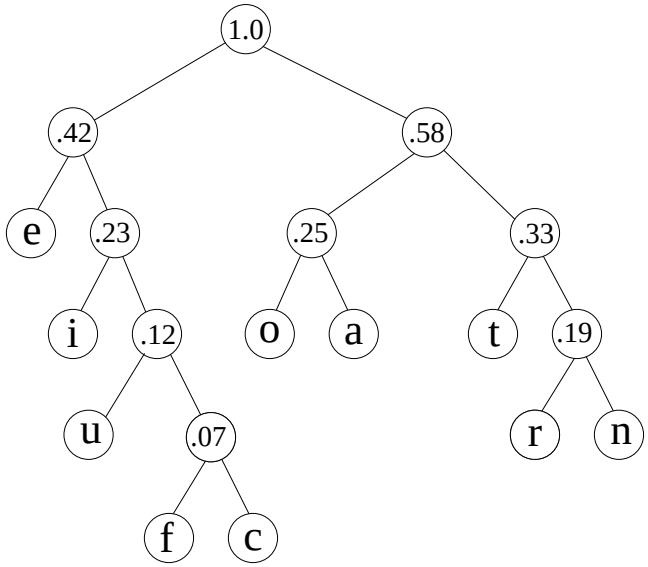
The edge set for the minimal spanning tree consists of edges:

.....

The weight of the minimal spanning tree is: .....

Question 9. (3 + 1 + 1 = 5 marks)

(a) Label the branches of the tree below to create a Huffman code and fill in the table below.



CHARACTER	CODE	CHARACTER	CODE
a		**n**	
c		**o**	
e		**r**	
f		**t**	
i		**u**	

(b) Encode: **a c u t e** .....

(c) Decode 0 1 1 1 0 1 1 1 0 1 0 1 0 1 1 1 1 1 1 0 0 1 0 1 0 1 0 0 1 1 1 1

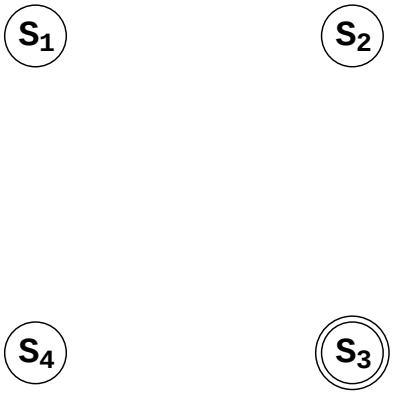
.....

Question 10. (4 + 2 = 6 marks) Consider the finite state machine given by the transition table below.

input→ / state ↓	0	1
S_1	S_4	S_2
S_2	S_3	S_3
S_3	S_4	S_3
S_4	S_4	S_4

initial state: S_1
acceptor state: S_3

(a) (i) Complete the state diagram below.



(ii) Describe the language accepted by this machine.

LANGUAGE ACCEPTED IS:

(b) Construct a finite state machine which accepts only the following two words: 110, 1101.

Question 11. (2 + 3 + 5 = 10 marks)

(a) How many different arrangements of the characters CALCULATION are there?
Briefly explain your answer.

ANSWER AND EXPLANATION:

(b) What is the coefficient of $x^2y^3z^4$ in the expansion of $(2x - y + 3z)^9$?

ANSWER:

(c) This question deals with bit strings, that is, strings of 0's and 1's. Complete the following table.

	Length: $\mathbf{n = 2}$	Length: $\mathbf{n = 5}$	Length: $\mathbf{n}$
How many bit strings of length . . . ?			
How many contain no 1's ?			
How many contain two 1's ?			
How many contain three 1's ?			
How many are palindromes ?			

A palindrome is a string or word that reads backward the same as forward,
eg 0000 and 10101 and 101101 are three palindromes. For reasons of simplicity, the third column will use $\mathbf{n}$ to be greater than or equal to three.

Question 12. (4 marks)

This question deals with Finite State Machines.

INPUT LANGUAGE: Strings of 0's and 1's

LANGUAGE TO BE ACCEPTED: Strings which consist of three 1's or six 1's or nine 1's, and so on. In fact any string of 1's with the number of 1's being a multiple of 3.

The language to be accepted written in symbolic notation is: $(111)^+$ or $(111)^n : n \geqslant 1$

Create a finite state machine which accepts the above language.

ANSWER: