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DISCUSSION PAPERS

Xiangkang Yin

Series A
00.11                                                   2000

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TWO-PART TARIFF COMPETITION IN DUOPOLY

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Abstract – Built on the location model, this paper studies the rivalry of two firms in an industry through two-part tariffs. It is found that kinky profit functions are responsible for the coincidence of imperfectly competitive equilibrium and cartelization outcome. A duopoly likely results in higher entry fees and industry profits and lower net consumers’ surplus than a monopoly because each duopolist has a smaller market size than the monopolist. But social welfare in the monopoly is lower than in the duopoly. In comparison with uniform pricing, two-part tariffs tend to have lower prices, more profits and welfare but the magnitude of net consumers’ surplus is ambiguous.

JEL Classification: D21, D42, D43, L11-L13

Keywords: Monopoly, Oligopoly, Two-Part Tariff, Cartelization

* The author would like to thank Chongwoo Choe and Gillian Hewitson for helpful discussions.
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1. INTRODUCTION

Since the seminal contribution of Oi (1971), the effects of two-part tariffs on price strategy and social welfare have long been interests of economists. One stream of the research concentrates on two-part tariffs in monopoly and their impacts on market efficiency and welfare (see Armstrong (1999), Auerbach and Pellechio (1978), Cassou and Hause (1999), Ng and Weisser (1974), Schmalensee (1981), Shaffer (1987), Vohra (1990)), and regulated monopoly in particular (see De Borger (2000), Panzar and Sibley (1989), Sherman and Visscher (1982), Vogelsang 1989)). Another stream studies two-part tariffs in a context of multiple firms and general equilibrium, where one or many monopolistic industries impose two-part tariffs and remaining competitive industries set linear prices (see Brown and Heal (1980), Brown et al. (1992), Clay (1994), Edlin and Epelbaum (1993), Edlin et al. (1998)). The third stream investigates two-part tariffs in oligopoly, including Gabrielsen and Sorgard (1998), Hurley (1996), Ziss (1995). These authors focus on such a market structure that an upstream monopolist sells her product to one or more downstream firms through two-part tariffs. An exception of this stream is Young (1991), which analyzes two-part tariffs of two monopolistic sellers with demand complementary because of transaction costs.

However, recent deregulation in telecommunication and other utilities has converted traditional monopolistic two-part tariffs into imperfectly competitive two-part tariffs since more than one company are now allowed to provide services in an industry. Elsewhere, head to head competition of two-part tariffs directly towards final consumers also prevails. For instance, stock brokers charge an annual maintenance fee on an account and a per-trade fee for each stock exchange; various clubs of heath, golf, book and wine, and many websites levy a membership fee and a per-use or per-unit price. In all these circumstances, competitors in the same business area provide close substitutes of products or services to well informed consumers. So, what is the Nash equilibrium of two-part tariff competition in these markets? Can such equilibrium be the first-best in social welfare? How two-part tariffs in
imperfect competition differ from those in monopoly and in cartelization? Are two-part tariffs better than uniform pricing in terms of consumers surplus, profits and social welfare? The purpose of this paper is to address these issues, joining in other research of imperfect competition with nonlinear pricing in line with Corts (1998) and Holmes’ (1989) work on third-degree price discrimination in the form of price discount, and Katz (1984) and Stole’s (1995) analysis of second-degree price discrimination with imperfect information and uncertainty.

The model of this paper is built on the location model of Hotelling (1929), Lancaster (1979) and Salop (1979), where a consumer purchases only one type of product produced by either firm in a duopolistic industry and allocates his remaining income to outside goods. The product can be attractive to a consumer only when it provides him more surplus (net of purchase cost and entry fee) than the rival product and outside goods do. Given two products’ per-unit prices, each firm’s lump-sum entry fee is not only constrained by the consumer’s outside choice, as analyzed widely in monopolistic two-part tariffs, but also constrained by the competitor’s entry fee. Which constraint is binding depends on whether the marginal consumer’s net surplus of purchasing the rival’s product is smaller than that of purchasing neither firm’s product. Thus, each firm’s profit function has a kink in the shift of constraints, which makes them stick to a particular entry fee in response to a range of the rival’s entry fee choices. This property of the model induces the equilibrium of two-part competition to be exactly the same as the outcome of two-part tariff cartelization under certain fairly plausible conditions. The per-unit price of each product under imperfect competition can be too high or too low from the viewpoint of social welfare. But if the marginal consumer’s demand for a good is equal to the average demand for that good, the price is equal to the marginal cost and is socially optimal. If this is the case, then symmetric equilibrium maximizes the social welfare. Surprisingly, the model suggests that consumers likely have to pay a higher entry fee in duopoly than in monopoly. Consequently, a two-part tariff duopoly may lead to lower net consumers’ surplus and more industry profits than a two-

1 But we focus on price and entry fee competition with given location of firms.
part tariff monopoly. The reason for these observations is that a duopolistic firm has a smaller market size in terms of the number of consumers than a monopolistic firm. Hence, the marginal consumer in the duopoly has higher consumer’s surplus than his counterpart in the monopoly, which in turn allows duopolistic firms to levy a higher entry fee. In comparison with uniform pricing competition, the two examples of the model show that two-part tariffs result in lower per-unit prices, more profits and social welfare but the relative magnitude of net consumers’ surplus is ambiguous.

The rest of the paper is organized as follows. Section II presents a model of two-part tariff competition and characterizes the properties of equilibrium and social welfare. Then it compares two-part tariff competition with two-part tariff monopoly and cartelization, and uniform pricing competition. Two examples in Section III illustrate the closed form of the equilibrium and provide a vehicle facilitating more detailed analysis. The final section discusses the two examples and draws concluding remarks.

2. A Model of Two-Part Tariff Competition

2.1) The Model and Equilibrium

Consider an industry of two close substitutes, x and y, supplied by two firms at per-unit prices, \( p_x \) and \( p_y \), with lump-sum entry fees, \( e_x \) and \( e_y \), respectively. Causal observations show that most customers purchase only one firm’s product or service to save entry fee. For instance, most families have one number of wired telephone but can make as many phone calls as they want. By this consideration, it is assumed that each individual consumer pays only one entry fee to purchase either type of goods supplied by the firms. This assumption is not restrictive because if a consumer likes to have two or more simultaneous entries, say, to have two telephone lines, we can think of him as two separate consumers as far as he does not generate extra utility from the increase in consumption variety and from the interaction of simultaneous access of the two goods. Consumers have diversified tastes of the goods and are indexed by taste parameter \( t \), distributing on [0, 1] with density and cumulative distribution functions \( f(t) \) and \( F(t) \). Let us first consider the case where only one good, say,
the \( x \)-good, is available. Assuming quasilinear utility, \( U_x(x, z, t) = u_x(x, t) + z \), where \( z \) is the amount of the numeraire good consumed, the indirect utility function of a consumer facing a price \( p_x \) with income \( m(t) \) can be written as \( v_x(p_x, t) + m(t) \), where \( v_x(p_x, t) = 0 \) if the price of good \( x \) is so high that the consumer does not want to consume it. In other words, the consumer, facing an entry fee \( e_x \), purchases good \( x \) if and only if \( v_x(p_x, t) + m(t) - e_x \geq m(t) \).

Roy’s Identity indicates that if the consumer purchases the \( x \)-good his demand is
\[
x(p_x, t) = -\frac{\partial v_x(p_x, t)}{\partial p_x}.
\]

However, if the consumer has two substitutes to choose, he needs to compare further the values of indirect utility from either consumption, i.e., comparing \( v_x(p_x, t) + m(t) - e_x \) with \( v_y(p_y, t) + m(t) - e_y \). So, the consumer chooses to buy the \( x \)-good if and only if
\[
v_x(p_x, t) - e_x \geq v_y(p_y, t) - e_y \quad \text{and} \quad v_x(p_x, t) - e_x \geq 0.
\]

These inequalities ensure that purchasing the \( x \)-good is more valuable than purchasing the \( y \)-good and purchasing the \( z \)-good only, respectively. To simplify notations, it is assumed that consumers are monotonically distributed in the sense that a consumer with a larger taste parameter \( t \) has a lower level of utility from the consumption of good \( x \) but a higher level of utility from the consumption of good \( y \); that is, \( \partial v_x(p_x, t)/\partial t < 0 \) and \( \partial v_y(p_y, t)/\partial t > 0 \). Let \( T_x \) be the marginal consumer of good \( x \), who is indifferent between staying the \( x \)-good market or not. Then \( T_x \) must be the smaller solution of following two equations:
\[
\begin{align*}
v_x(p_x, t) - e_x &= v_y(p_y, t) - e_y \\
v_x(p_x, t) &= e_x.
\end{align*}
\]

These equations imply that the marginal consumer is indifferent either between purchasing the \( x \)-good and \( y \)-good or between consuming the \( x \)-good and not consuming the \( x \)- and \( y \)-goods. The marginal consumer of good \( y \), \( T_y \), is defined similarly. By the monotonic distribution assumption, there is \( T_x \leq T_y \); that is, the marginal consumer of good \( x \) more or at least not less prefers the \( x \)-good to the \( y \)-good than the marginal consumer of good \( y \). When the marginal consumer of good \( x \) differs from that of good \( y \), i.e., \( T_x < T_y \), the maximum entry fees can be determined by \( e_x = v_x(p_x, T_x) \) and \( e_y = v_x(p_y, T_y) \). On the other hand, if the two marginal consumers overlap, i.e., \( T_x = T_y = T \), there must be \( v_x(p_x, T) - e_x = v_y(p_y, T) - e_y \) and
\( e_x \leq v_x(p_x, T), \) \( e_y \leq v_y(p_y, T). \) By these considerations, it is clear the maximum entry fee firm \( x \) can set is
\[
e_x = \min\{v_x(p_x, T_x), v_x(p_x, T_x) - v_y(p_y, T_x) + e_y\}. \tag{2}
\]
A similar formula is applicable to firm \( y. \) Normalizing the total population to unity, the total demands for goods \( x \) and \( y \) are, respectively,
\[
X(p_x, T_x) = \int_0^{T_x} x(p_x, t) f(t) dt \quad \text{and} \quad Y(p_y, T_y) = \int_{T_x}^1 y(p_y, t) f(t) dt,
\tag{3}
\]
where each individual’s demands \( x(p_x, t) \) and \( y(p_y, t) \) are available through applying Roy’s Identity to the indirect utility functions as mentioned earlier.

The production is assumed to have a constant marginal cost \( c_x \) and \( c_y, \) respectively, but incurs no fixed costs. Therefore, the profit functions of the two firms can be written as
\[
\pi_x = e_x F(T_x) + (p_x - c_x)X(p_x, T_x) \quad \text{and} \quad \pi_y = e_y [1 - F(T_y)] + (p_y - c_y)Y(p_y, T_y). \tag{4}
\]
The goal of the firms in two-part tariff competition is to choose their tariffs \( (p_x, e_x) \) and \( (p_y, e_y), \) respectively, to maximize their own profits. Equation (2) shows that in two-part-tariff competition, firms impose mutual constraints on their magnitudes of lump-sum entry fees. If only one firm, say, firm \( x, \) in the market, it can charge the maximum entry fee \( v_x(p_x, T_x). \) However, when firm \( y \) is also in the market, it typically has to charge a lower entry fee because of competition. Let \( T_x \) be the marginal consumer by whom firm \( x \) shifts its entry fee scheme from \( e_x = v_x(p_x, T_x) \) to \( e_x = v_x(p_x, T_x) - v_y(p_y, T_x) + e_y, \) so it can be determined by
\[
\bar{T}_x = \arg\{e_y = v_y(p_y, T_x)\}. \tag{5}
\]
Note, \( \bar{T}_x \) also indexes the consumer who is indifferent between consuming the \( y \)-good and consuming neither the \( x \)-good nor \( y \)-good. Thus, (2) can also be rewritten as
\[
e_x = \begin{cases} v_x(p_x, T_x) & \text{if } T_x \leq \bar{T}_x, \\ v_x(p_x, T_x) - v_y(p_y, T_x) + e_y & \text{if } T_x \geq \bar{T}_x. \end{cases} \tag{6}
\]
It is possible that \( \bar{T}_x \leq 0 \) or \( \bar{T}_x \geq 1, \) which means that firm \( x \) sets entry fee \( e_x = v_x(p_x, T_x) - v_y(p_y, T_x) + e_y \) or \( e_x = v_x(p_x, T_x) \) for all \( T_x \in [0, 1]. \) Equation (6) indicates that \( e_x \) is uniquely determined by \( T_x \) given \( p_x, p_y, e_y. \) In other words, the decision of choosing \( (p_x, e_x) \) is equivalent to choosing \( (p_x, T_x). \) The same is applicable to firm \( y. \) Substituting (6) into (4), we
can see that given the firm y’s decision on \( p_y \) and \( e_y \), the profit function of firm x is continuous in both \( p_x \) and \( e_x \) that

\[
\pi_x = \pi_{x1} = v_x(p_x, T_x)F(T_x) + (p_x - c_x)X(p_x, T_x) \quad \text{if } T_x \leq \bar{T}_x, \tag{7}
\]

\[
\pi_x = \pi_{x2} = [v_x(p_x, T_x) - v_y(p_y, T_x + e_y)]F(T_x) + (p_x - c_x)X(p_x, T_x) \quad \text{if } T_x \geq \bar{T}_x. \tag{8}
\]

When \( \bar{T}_x \geq 1 \) or \( \bar{T}_x \leq 0 \), the profit function has only one branch that \( \pi_x = \pi_{x1} \) or \( \pi_x = \pi_{x2} \) on \( T_x \in [0, 1] \). It is worthwhile to note that on the branch \( \pi_x = \pi_{x1} \) (excluding the point \( T_x = \bar{T}_x \)) the marginal consumers of good x and good y are different, i.e., \( T_x < T_y \), while on the branch \( \pi_x = \pi_{x2} \) the two marginal consumers are the same consumer, i.e., \( T_x = T_y \). Taking the partial derivative of profit function (7) or (8) with respect to \( p_x \) yields the first-order condition (FOC)

\[
-x(p_x, T_x)F(T_x) + X(p_x, T_x) + (p_x - c_x)\partial X(p_x, T_x)/\partial p_x = 0. \tag{9}
\]

On the other hand, the partial derivative with respect to \( T_x \) leads to

\[
\partial \pi_x/\partial T_x = \partial \pi_{x1}/\partial T_x = F(T_x)\partial v_x(p_x, T_x)/\partial T_x + [v_x(p_x, T_x) + (p_x - c_x)x(p_x, T_x)]f(T_x) \quad \text{if } T_x \leq \bar{T}_x, \tag{10}
\]

\[
\partial \pi_x/\partial T_x = \partial \pi_{x2}/\partial T_x = [\partial v_x(p_x, T_x)/\partial T_x - \partial v_y(p_y, T_x)/\partial T_x]F(T_x) + [v_x(p_x, T_x) - v_y(p_y, T_x) + e_y]f(T_x) + (p_x - c_x)x(p_x, T_x)f(T_x) \quad \text{if } T_x \geq \bar{T}_x. \tag{11}
\]

Similar derivatives are applicable to firm y. It is plausible to assume that \( \pi_{x1} \) and \( \pi_{x2} \) are concave in \( T_x \), and (10) and (11) have a zero point, \( T_{x1} \) and \( T_{x2} \), respectively, maximizing each branch of the profit function. However, it is quite likely that \( T_{x1} \) does not fall within the interval \((0, \bar{T}_x)\) and \( T_{x2} \) not within \((\bar{T}_x, 1)\). Thus, given firm y’s decision of price and entry fee, the possible candidates for firm x’s optimal reaction in determining the marginal consumer is to choose \( T_x \) at 0, 1, \( T_{x1} \), \( T_{x2} \) or \( \bar{T}_x \) as far the last three terms are in interval \((0, 1)\).

A further comparison between profit values at these points can determine the best response in the choice of marginal consumer. The detailed process will be given in Example 1 below.

Equilibrium concept in this model is Nash equilibrium. To facilitate discussion below, I define several definitions here. A full-cover equilibrium means that each consumer is served by a firm while in a partial-cover equilibrium some consumers are not served by either firm. A symmetric equilibrium is that both firms set the same per-unit price and entry fee and have the same market size so that \( p_x = p_y \), \( e_x = e_y \) and \( T_x = 1 - T_y \). By this definition,

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2 Market size is defined as the number of clients a firm has.
we have \( T_x = T_y = \frac{1}{2} \) in full-cover symmetric equilibrium. There are several conclusion can be drawn immediately from (9)-(11).

**Proposition 1.** (i) If the demand of marginal consumer is equal to the average demand, i.e., \( x(p_x, T_x) = X(p_x, T_x)/F(T_x) \), the equilibrium per-unit price is equal to marginal cost. (ii) Equilibrium \( T_x = 1 \) or \( T_x = 0 \) implies that either firm \( y \) or firm \( x \) exits from the market so that the market is monopoly. (iii) If equilibrium realizes at \( T_x = T_{x1} \), it is a partial-cover equilibrium provided \( T_{x1} \neq \bar{T}_x \) and there must exist \( T_x < \bar{T}_x \) and \( T_{x1} < T_{y1} \). (iv) If equilibrium rests at \( T_x = T_{x2} \) or \( T_x = \bar{T}_x \), it is a full-cover equilibrium and there is \( T_{x2} = T_{y2} \) or \( T_x = \bar{T}_y \).

**Proof:** (i) and (ii) are obvious. For (iii), we need only to note that \( T_x = T_{x1} \) is the maximum point of \( \pi_{x1} \) and firm \( y \) does not impose any constraint on firm \( x \) on this branch of profit function so that the two firms marginal consumers are separate. To (iv), first note that \( T_x = T_{x2} \) is the maximum point of \( \pi_{x2} \) and the two marginal consumers on this branch of profit are coincident. Second, if the equilibrium is \( T_x = \bar{T}_x \) and \( T_y = \bar{T}_y \) we have \( e_x = v_x(p_x, \bar{T}_x) \) and \( e_y = v_y(p_y, \bar{T}_y) \) by the first line of equation (6) and its counterpart. Substituting these values into the second line of (6) yields \( \bar{T}_x = \bar{T}_y \).

It is important to note that in equilibrium \( T_x = T_{x1}, T_y = T_{y1} \) or \( T_x = T_y = \bar{T}_x = \bar{T}_y \) each firm charges an entry fee equal to its marginal consumer’s surplus, \( v_x(p_x, T_x) \), which implies that competition does not reduce the entry fees in these equilibria. This property of two-part tariff competition has a significant implication on collusive outcome of noncorporative duopolistic firms and we will discuss further below. The reason for such a high entry fee is that when the market size of firm \( x \) is smaller than \( \bar{T}_x \), the firm is virtually “a monopolist” in its market and the rival’s price and entry fee do not impose any restriction on its decision-making. The only reason for a consumer to leave its market is the consumption of outside goods. So, firm \( x \) can charge entry fee up to \( v_x(p_x, T_x) \) without a reduction of market size.

### 2.2) Welfare Analysis of Equilibrium

The welfare function is defined as the sum of consumers’ surplus and firm profits:
\[ W(p_x, p_y, T_x, T_y) = \int_0^T v_x(p_x, t) f(t) dt + \int_1^T v_y(p_y, t) f(t) dt + (p_x - c_x) X(p_x, T_x) + (p_y - c_y) Y(p_y, T_y) \]  

(12)

Its partial derivative with respect to \( p_x \) and \( T_x \) are

\[
\frac{\partial W(p_x, p_y, T_x, T_y)}{p_x} = (p_x - c_x) \frac{\partial X(p_x, T_x)}{\partial p_x}, \quad \text{if } T_x < T_y \tag{13}
\]

\[
\frac{\partial W(p_x, p_y, T_x, T_y)}{T_x} = \left[ v_x(p_x, T_x) + (p_x - c_x) x(p_x, T_x) \right] f(T_x), \quad \text{if } T_x < T_y \tag{14}
\]

\[
\frac{\partial W(p_x, p_y, T_x, T_y)}{T_x} = \left[ v_x(p_x, T) - v_y(p_y, T) + (p_x - c_x) x(p_x, T) - (p_y - c_y) y(p_y, T) \right] f(T).
\]

(15)

Evaluating (13) at two-part-tariff competition equilibrium yields 

\[
\frac{\partial W(p_x, p_y, T_x, T_y)}{p_x} = \left[ v_x(p_x, T_x) - x(p_x, T_x) \right] f(T_x) = X(p_x, T_x),
\]

which can be either positive or negative depending on whether the demand of marginal consumer is greater or smaller than the average demand. If the demand of marginal consumer is equal to the average demand, i.e., \( x(p_x, T_x) = X(p_x, T_x) \), the per-unit price in equilibrium is equal to marginal cost and maximizes welfare.

**Proposition 2.** The equilibrium per-unit price of each good under two-part tariff competition can be too high or too low from the social welfare point of view. But if the demand of marginal consumer is equal to the average demand it maximizes social welfare.

Turning to market size and entry fees, we have

**Proposition 3.** If two-part tariff competition leads to a partial-cover equilibrium, the market size is too small and entry fees are too high for welfare optimum. If the market is fully covered in equilibrium, the market size is socially optimal but the determination of marginal consumer may not.

**Proof:** Recalling (10), the value of (14) at partial-cover equilibrium \((p_x, p_y, T_x1, T_y1)\) is

\[
\frac{\partial W(p_x, p_y, T_x1, T_y1)}{\partial T_x} = -F(T_x1) \frac{\partial v_x(p_x, T_x1)}{\partial T_x} > 0,
\]

which implies social welfare can be improved if each firm lowers the entry fee and enlarges the market size. On the other hand, for a full-cover equilibrium, all consumers are serviced under two-part tariff competition so the market size reaches social optimum. However, recalling (11), the value of (15) at the full-cover equilibrium is not necessarily to be zero in general.
The first part the proposition resembles the conclusion obtained in monopoly (see Varian (1989)). This is not surprising because as we indicated above that duopolistic firms are virtually “monopolists” in their market ranges in the partial-cover equilibrium so that they behave like a monopolist. For the full-cover equilibrium, we can reach much stronger conclusion if we assume symmetry.

**Proposition 4.** If indirect utility and marginal costs are symmetric that \( v_x(p, \frac{1}{2} - t) = v_y(p, \frac{1}{2} + t) \) and \( c_x = c_y \), then a symmetric full-cover equilibrium determines a socially optimal marginal consumer. Moreover, if the demand of marginal consumer is equal to the average demand, the equilibrium maximizes social welfare.

**Proof:** Substituting \( p_x = p_y \) and \( T = \frac{1}{2} \) into (15) yields \( \partial W/T = 0 \), which gives the first conclusion. Recalling Proposition 2, the second conclusion is immediate.

To certain extent, this preposition resembles the outcome of Bertrand price competition in a homogenous duopoly that two firms are enough to obtain the first-best social welfare through competition. However, Bertrand equilibrium leads to zero profits and maximum consumers’ surplus while in two-part tariffs firms typically earn positive profits and consumers’ net surplus is not maximized. Moreover, when the market is fully covered and the marginal consumer is determined, entry fees are only a wealth transfer from consumers to producers and do not affect social welfare. Hence, the symmetric full-cover equilibrium with marginal-cost pricing is only one of social optima.

### 2.3) Comparison with Monopoly

To study the effects of competition, it is worthwhile to compare duopolistic two-part tariffs with monopolistic two-part tariffs. The monopoly to be compared is the situation, where only one firm, say, firm \( x \), in the market but the utility and demand for the \( x \)-good is the same as that in the duopoly. This assumption makes the comparison between monopoly and duopoly in a common ground since the demand for the \( x \)-good is identical, independent of the presence of good \( y \) as far as consumers enter the \( x \)-good market. For such a monopolist, FOC for per-unit price determination is the same as (9) (see Varian (1989)). The
real difference appears in the determination of marginal consumer. In the monopoly case the
market size is determined by the zero point of (13), \( T_{x1} \), without constraint \( T_{x1} \leq \bar{T}_x \) while in
the duopoly case it can be either \( T_{x1} \) or \( \bar{T}_x \) or \( T_{x2} \).

**Proposition 5.** Each firm’s market size in duopoly is not larger than a monopolist’s
market size but the total market size in duopoly is larger than or equal to market size of the
monopoly.

**Proof:** Rewrite (11) to find that its zero point is characterized by

\[
F(T_x)\frac{\partial v_x(p_x, T_x)}{\partial T_x} + [v_x(p_x, T_x + (p_x - e_x)x(p_x, T_x))]f(T_x) = F(T_x)\frac{\partial v_y(p_y, T_x)}{\partial T_x} + [v_y(p_y, T_x) - e_y]f(T_x). \quad (16)
\]

Since the downward slopping left-hand side of (16) is the same as the right-hand side of (10)
and the right-hand side of (16) is positive, it is not hard to find that \( T_{x1} > T_{x2} \). Moreover, if \( \bar{T}_x \)
is a duopoly equilibrium, it requires that that \( \pi_{x1} \) increases on the left of \( \bar{T}_x \) and \( \pi_{x2} \) decreases
on the right, which implies that \( T_{x1} > \bar{T}_x > T_{x2} \). As possible market size of a single firm in a
duopoly equilibrium is \( T_{x1} \) or \( \bar{T}_x \) or \( T_{x2} \), it does not larger than a monopolist’s market size of
\( \min\{T_{x1}, 1\} \).

In terms of total market size, if the marginal consumer of good \( x \) in the duopoly is \( T_{x1} \),
then firm \( x \) has the same market size in either the monopoly or duopoly. So, adding
consumers of good \( y \), the duopoly definitely has a larger total market size. On the other hand,
if the marginal consumer in the duopoly is \( T_x = \bar{T}_x \) or \( T_x = T_{x2} \), then consumers with tastes \( t \in
\{T_x, \min\{T_{x1}, 1\}\} \) definitely purchase the \( y \)-good since their reason of leaving the \( x \)-good
market is simply more net surplus from purchasing the \( y \)-good. So, including other
consumers in the \( y \)-good market, the total market size in the duopoly cannot be smaller than
the monopolistic market size.

\( \smile \)

It should be cautious about the difference between duopoly and monopoly prices. As
mentioned at the beginning of this subsection, the FOCs determining the prices are the same
(equation (9)) under two different market structures. However, the market sizes of a single
firm under two market structures are typically different, which induce different per-unit
prices. Moreover, because a monopolistic firm may have a larger market size than a
duopolistic firm, the marginal consumer likely has less consumer’s surplus (before entry fee) in the monopoly than he can have in the duopoly, provided the price difference under two market structures is not too large. Since entry fees are constrained by the marginal consumer’s surplus, it is quite possible that the monopoly entry fee is lower than the duopoly entry fee. We will turn to this point in more detail in the examples below.

2.4) Comparison with Cartelization

When two firms in an industry form a cartel, they coordinate their strategy in setting per-unit prices and entry fees to maximize industry profits,

$$\Pi = e_x F(T_x) + (p_x - c_x)X(p_x, T_x) + e_y [1 - F(T_y)] + (p_y - c_y)Y(p_y, T_y).$$  \hspace{1cm} (17)

The constraint (2) on \(e_x\) and its counterpart on \(e_y\), and \(T_x \leq T_y\) must still be satisfied. As mentioned before, when \(T_x < T_y\), the constraint (2) and its counterpart collapse to \(e_x = v_x(p_x, T_x)\) and \(e_y = v_y(p_y, T_y)\). Substituting them into (17) and taking partial derivatives with respect to \(p_x\) and \(p_y\), \(T_x\) and \(T_y\), respectively, yield the same formulas as (9) and (10), and their counterparts for firm \(y\). Now, consider four FOCs: (9), the right-hand side of (10) equal zero, and their counterparts. If the solution of these FOCs leads to \(T_{x1} < T_{y1}\), it maximizes joint profits of the industry. This implies the proposition below.

**Proposition 6.** If two-part tariff competition results in a partial-cover equilibrium, then it coincides with a cartelization outcome.

However, if the above solution turns out to be \(T_{x1} \geq T_{y1}\), it implies that it is better for the cartel to supply all consumers. Then, we have \(T_x = T_y = T\). As far as the marginal consumer is determined, the cartel will levy as high entry fees as possible, so that it sets the entry fees equal to the consumer surplus of marginal consumer, \(e_x = v_x(p_x, T)\) and \(e_y = v_y(p_y, T)\). Substituting them into (17) to obtain

$$\Pi = v_x(p_x, T)F(T) + (p_x - c_x)X(p_x, T) + v_y(p_y, T)[1 - F(T)] + (p_y - c_y)Y(p_y, T).$$  \hspace{1cm} (18)

Thus, the independent decision variables for the cartel are \(p_x\), \(p_y\) and \(T\). The partial derivatives with respect to \(p_x\) and \(p_y\) provide the same FOCs as (9) and its counterpart but the partial derivative with respect to \(T\) leads to
\[
\begin{align*}
[\partial v_x(p_x, T)/\partial T - \partial v_y(p_y, T)/\partial T]F(T) + \partial v_y(p_y, T)/\partial T + \\
[v_x(p_x, T) - v_y(p_y, T) + (p_x - c_x)x(p_x, T) - (p_y - c_y)y(p_y, T)]f(T) &= 0. 
\end{align*}
\]

Comparing the solution of (19) with the zero point of (11) or \( \bar{T}_x = \bar{T}_y \), we can see that the marginal consumer in cartelization may differ from that of duopolistic competition in full-cover equilibrium. In the former the cartel takes the effects of marginal consumer change on both firms’ profits into account while in the latter each firm only considers its own profits. However, it is possible that the marginal consumers are the same in the two industry structures under certain conditions, such as in symmetric equilibria. But it should be noted that even in such equilibria, their entry fees might still differ. In a non-trivial special case, the full-cover symmetric equilibrium is a cartelization outcome.

**Proposition 7.** If indirect utility functions, consumer distribution and marginal costs are symmetric, then symmetric full-cover equilibrium \( \bar{T}_x = \bar{T}_y = \frac{1}{2} \) of two-part tariff competition leads to an outcome of cartelization.

*Proof:* The symmetry implies \( T = \frac{1}{2} \) is the zero point of (19) so that the marginal consumers are the same under two market structures and so do the per-unit prices. On the other hand, two-part tariff competition reaching equilibrium at \( \bar{T}_x = \bar{T}_y = \frac{1}{2} \) implies that firms charge entry fees, \( e_x = v_x(p_x, \frac{1}{2}) \) and \( e_y = v_y(p_y, \frac{1}{2}) \). Apparently, those are also the entry fees the cartel wants to charge.

The similarity between two-part tariff competition and cartelization seems anti-intuitive as we generally expect competitive outcomes should differ from collusive outcomes. But a closer look on Propositions 6 and 7 shows that their intuitions are straightforward. Our model is similar to conventional location model of Salop (1979) in that each firm has a monopoly range. In partial-cover equilibrium both firms operate in their monopoly ranges and one firm’s price and entry fee do not affect the demand for the other firm’s product. Thus, the outcome of two “monopolistic” firms is equivalent to that of one cartel. Turning to Proposition 7, the symmetry makes both the imperfectly competitive duopoly and the cartel select the same marginal consumer. When \( \bar{T}_x = \bar{T}_y = \frac{1}{2} \), the marginal consumer is indifferent

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3 Symmetric distribution means \( f(\frac{1}{2} - t) = f(\frac{1}{2} + t) \) so that \( F(\frac{1}{2}) = \frac{1}{2} \).
between consuming good \( x \) and consuming good \( y \) and consuming no goods \( x \) and \( y \) so that both firms’ profit functions kink at \( t = \frac{1}{2} \). At this kink, either a marginal reduction in entry fee to steal the rival’s clients or a marginal increase in entry fee at the expense of smaller market size does not raise a firm’s profit. So, the firms stay in the kink, where the rival’s per-unit price and entry fee just begin to impose a constraint on a firm’s entry fee, and both firms still can set an entry fee equal to the marginal consumer’s surplus as the cartel always does.

### 2.5) Comparison with Uniform Pricing

Linear prices can be considered as a special case of two-tart tariffs that the entry fees are restricted to zero. Similar to (1), the marginal consumer is determined by

\[
T_x = \min\{h(p_x, p_y), 1\},
\]

where

\[
h(p_x, p_y) = \arg_{T_x}\{v_x(p_x, T_x) = v_y(p_y, T_x)\}.
\]

Therefore, given \( p_y \), (20) implicitly determines \( T_x \) as a function of \( p_x \). Let \( p_x = \max\{\arg_{p_x}\{v_x(p_x, 1) = v_y(p_y, 1)\}, 0\} \), the derivative of \( T_x \) with respect to \( p_x \) may discontinues at \( p_x \) since

\[
\frac{dT_x}{dp_x} = 0 \quad \text{if} \quad p_x \leq p_x,
\]

\[
\frac{dT_x}{dp_x} = x(p_x, T_x)\left[\frac{\partial v_x(p_x, T_x)}{\partial T_x} - \frac{\partial v_y(p_y, T_x)}{\partial T_x}\right] \quad \text{if} \quad p_x \geq p_x.
\]

Because the profit function of firm \( x \) under uniform pricing is \( \pi_x = (p_x - c_x)X(p_x, T_x) \), recalling (3) its derivative with respect to \( p_x \) is

\[
\frac{\partial \pi_x}{\partial p_x} = X(p_x, T_x) + (p_x - c_x)\frac{\partial X(p_x, T_x)}{\partial p_x} + (p_x - c_x)x(p_x, T_x)f(T_x)dT_x/dp_x.
\]

Substituting (21) or (22) into (23) and setting it equal to zero yield two zero points, \( p_{x1} \) and \( p_{x2} \). Since the profit function may kink at \( p_x \), firm \( x \)’s best price response to firm \( y \)’s price \( p_y \) can be either \( p_{x1} \) or \( p_{x2} \) or \( p_x \), depending on which results in the highest profit. It is clear that \( T_x = 1 \), and accordantly \( dT_x/dp_x = 0 \), implies that firm \( x \) is the sole supplier of the market. Hence, it is reasonable to assume that the first two terms on the right-hand side of (23) is positive but monotonically decreases before reaching zero. Noting the last term of (23) is negative if it not equal to zero, we find that \( p_{x1} > p_{x2} \). Moreover, if uniform-pricing
competition results in an equilibrium price \( p_{x1} \) or \( p_x \), it means firm \( y \) has been driven out of the market and firm \( x \) is a monopolist. Thus, for nondegenerated uniform-pricing competition, we need only to find out equilibrium \( p_{x2} \) and its counterpart \( p_{y2} \).

Comparing (23) with (9), it is clear that the equilibrium price of uniform pricing can be either higher or lower than that of two-part tariff competition. Without more specification of the model, it is impossible to draw unambiguous conclusions on prices and welfare effects. These results will be given in the two examples below.

3. Two Examples

To have a closer look on two-part tariff competition and explicitly characterize equilibrium in a closed form, I present two examples in this section. In both examples, consumers are uniformly distributed on \([0, 1]\) and two firms have the same marginal cost; i.e.,

\[ c_x = c_y = c, \]

3.1) Example 1

In this example, the indirect utility functions are in the additive form that

\[ v_x(p_x, t) = v(p_x) + a(1 - t) \quad \text{and} \quad v_y(p_y, t) = v(p_y) + at, \quad a \geq 0. \]  

(24)

To simplify notations, write \( v(c) \) as \( v \). Utility specification (24) implies that all consumers have the same basic indirect utility \( v(p_x) \) and \( v(p_y) \) from the consumption of goods \( x \) and \( y \). Their preference difference is characterized by the additive terms \( a(1 - t) \) and \( at \), similar to the transportation cost in the Hotelling model. When \( a = 0 \), it implies homogenous consumers. For the consumer with taste parameter \( t = 0 \) (\( t = 1 \)), the additional value of \( x \)-good consumption over \( y \)-good consumption (\( y \)-good consumption over \( x \)-good consumption) is equal to \( a \) if the prices of both goods are equal. On the other hand, for each good, the preference deference between consumers with tastes \( t = 0 \) and \( t = 1 \) is also equal \( a \). In this sense, \( a/v \) can be considered as a relative measure of product differentiation and/or consumer diversity. With such indirect utility functions, each consumer’s demand for the \( x \)-good is independent of taste parameter \( t \); i.e., \( x(p_x, t) = x(p_x) = -\partial v(p_x)/\partial p_x \) and the aggregate demand
is simply \( X(p_x, T_x) = x(p_x)T_x \). Since the demand of marginal consumer is equal to the average demand, FOC (9) leads to marginal cost pricing that \( p_x = p_y = c \). Substituting the special form of indirect utility functions (24) and \( p_x = p_y = c \) into (5), (10) and (11), we can obtain

\[
\tilde{T}_x = (e_y - v) / a, \ T_{x1} = (a + v) / 2a \quad \text{and} \quad T_{x2} = (a + e_y) / 4a.
\]

To find out the entry-fee reaction function of firm \( x \), we need to consider three cases:

(i) \( a/v \geq 1 \)

When \( e_y \in (0, (a + 4v)/3) \), there are \( \tilde{T}_x < 1, \ T_{x1} > \tilde{T}_x \quad \text{and} \quad T_{x2} \in (\tilde{T}_x, 1) \). The profit function either has only one branch \( \pi = \pi_{x2} \) if \( \tilde{T}_x \leq 0 \) or its left branch, \( \pi_{x1} \), monotonically increases on \((0, \tilde{T}_x)\) if \( \tilde{T}_x > 0 \). Thus the profit function reaches maximum at \( T_x = T_{x2} \) and the best response is to set entry fee at \( e_x = a - 2aT_x + e_y = (a + e_y) / 2 \).

When \( e_y \in ((a + 4v)/3, (a + 3v)/2) \), there are \( T_{x1} > \tilde{T}_x, \ T_{x2} < \tilde{T}_x \) and \( \tilde{T}_x \in (0, 1) \). The left branch of the profit function, \( \pi_{x1} \), monotonically increases while the right branch, \( \pi_{x2} \), monotonically decreases so that the maximum is at \( T_x = \tilde{T}_x \) and the best reaction of entry fee is \( e_x = a - 2aT_x + e_y = a + 2v - e_y \).

When \( e_y \in ((a + 3v)/2, +\infty) \), two branches of the profit function are monotonic so that profit maximum is at \( T_x = \tilde{T}_x \) and optimal entry fee is \( e_x = a + 2v - e_y \).

When \( e_y \in (0, (a + 4v)/3) \), the best reaction is \( T_x = T_{x2} \) and \( e_x = (a + e_y) / 2 \) as discussed above.

(ii) \( a/v \in (1/2, 1) \)

When \( e_y \in (0, (a + 4v)/3) \), \( a/v \in (1/2, 1) \), two branches of the profit function are monotonic so that profit maximum is at \( T_x = \tilde{T}_x \) and optimal entry fee is \( e_x = a + 2v - e_y \).

When \( e_y \in ((a + 4v)/3, a + v) \), the profit has only one monotonically increasing branch \( \pi = \pi_{x1} \). Then profit-maximizing response is \( T_x = 1 \) and \( e_x = v + a(1 - T_x) = v \).

(iii) \( a/v \in (0, 1/2) \)

When \( e_x \in (0, (a + v)) \), it is still true that either the profit function has only one branch \( \pi = \pi_{x2} \) or its left branch \( \pi_{x1} \) monotonically increases in \((0, \tilde{T}_x)\). However, it is possible that
\[ T_{x2} = \frac{(a + e_y)}{4a} > 1 \text{ if } e_y > 3a. \] Thus, optimal marginal consumer is \( T_x = \min\{(a + e_y)/4a, 1\} \) and the optimal entry fee should be \( e_x = a - 2aT_x + e_y = \max\{(a + e_y)/2, e_y - a\} \).

When \( e_y \in (a + v, +\infty) \), optimal response is \( T_x = 1 \) and \( e_x = v \) as discussed above. Cases (ii) and (iii) can be considered as a short version of case (i) in the sense that as \( a/v \) decreases some segments of the reaction curve in (i) has been chopped off. For firm \( y \), there is a similar reaction function. Figure 1 below illustrates these reaction curves in case (i).

Through these two reaction functions, routine calculation gives equilibrium market coverage and entry fee (see the Appendix for the derivation)\(^5\)

\[ T_x = T_y = \frac{1}{2}, \quad (25) \]
\[ e_x = e_y = \min\{a, v + a/2\}. \quad (26) \]

Thus, the market in this example is always fully covered, independent of the spread extensity of consumers, \( a \), and the magnitude of the basic indirect utility, \( v \). About entry fees there are two interesting observations. The first is that equilibrium entry fees tend to zero as \( a \) tends to zero. This demonstrates that if consumers are homogenous firms cannot charge two-part tariffs in the model. Second, the entry fees increase as the value of the basic indirect utility \( v \) is larger and/or consumers are more diversified (\( a \) is larger). This is obvious because as consumers value the commodity higher, they are willing to pay more so that firms can levy a higher entry fee. On the other hand, more diversified consumers implies firms have more market power and consequently they can set a higher entry fee. Since it is a full-cover symmetric equilibrium and per-unit price is equal to marginal cost, the equilibrium is socially optimal as proved by Proposition 4.

**Proposition 8.** With specification (24), two-part tariff competition results in a socially optimal outcome with marginal-cost pricing and entry fees given in (26).\(^6\)

---

\(^5\) Figure 1 and the Appendix show that when \( a/v > 2 \), there are a continuum of equilibria. I focus on the symmetric equilibrium in the text.

\(^6\) Actually, the results of marginal-cost pricing and (25)-(26) can be achieved by a more relaxed assumption of indirect utility functions that \( v_x(p_x, t) = v_x(p_x) + a(1 - t) \), \( v_y(p_y, t) = v_y(p_y) + at \) and \( v_x(c) = v_y(c) = v \).
Routine calculation shows that the net consumers’ surplus in equilibrium is
\[ 2 \int_0^{0.5} [v + a(1-t)] dt - \min\{a, v + a/2\} = \max\{v - a/4, a/4\}. \] (27)

As each firm’s income is the entry fee, the industry profit is equal to
\[ \min\{a, v + a/2\}. \] (28)

Social welfare, defined as the sum of net consumers’ surplus and industry profits, is
\[ 2 \int_0^{0.5} [v + a(1-t)] dt = v + 3a/4. \] (29)

Comparison with monopoly

If the market is supplied by a single monopolist, it also sets the per-unit price at the marginal cost as discussed in the previous section. But the market size and entry fee are different. The monopolist chooses the marginal consumer at the zero point of (10), \( T_{x1} \), or supplies the whole market so that the marginal consumer is
\[ T_x = \min\{(a + v)/2a, 1\}. \] (30)

This implies that the total market coverage in the monopoly is smaller than in the duopoly if consumers are substantially diversified \((a/v > 1)\), as demonstrated in Proposition 5. Turning to the entry fee, the monopolist wants it to be as high as the indirect utility of the marginal consumer, i.e.,
\[ e_x = v + a(1 - T_x) = \max\{(a + v)/2, v\}. \] (31)

Summarizing these discussions, we have

Proposition 9. If the indirect utility functions are those of (24), duopolistic firms set the same equilibrium per-unit prices as a monopolist does but charge higher equilibrium entry fees if and only if \(a/v \geq 1\).

Proof: Since the per-unit price is equal to marginal cost irrelevant the market structure, the first part of the proposition is immediate. For entry fees, it is obvious, recalling (26) and (31), that \(\min\{a, v + a/2\} > \max\{(a + v)/2, v\}\) if and only if \(a/v \geq 1\).

---

Coyte and Lindsey (1988) present a similar two-part tariff monopoly with linear demand.
This result of entry fees is quite surprising because when $a/v > 1$, the monopolist charges a lower entry fee than the duopolists. In other words, more firms enter competition may yield a higher entry fee to consumers, which is not so straightforward from the conventional standpoint. The reason for such a result is that in the duopoly the marginal consumer is in the middle of distribution support $[0, 1]$, who has the indirect utility $v + a/2$ in equilibrium. Although competition may lead to the entry fees lower than this amount, the difference is not substantial. From (26) we can see that when $a/v \in [0, 2]$, the difference is $v + a/2 - a = a/2 - v$, which monotonically decreases to zero as $a/v$ increases to 2. When $a/v \in (2, +\infty)$, the difference is zero. On the other hand, the profit-maximizing monopolist always supplies more than a half of consumers since (30) shows that $T_x > \frac{1}{2}$ holds in the monopoly market. Thus, the monopolist always sets an entry fee below $v + a/2$ so that when $a/v \geq 1$, the entry fee in the monopoly is lower than that in the duopoly. Because of lower entry fee, the monopoly does not always make consumers worse off relative to the duopoly.

**Proposition 10.** If the indirect utility functions are those of (24), a duopoly yields less net consumers’ surplus than a monopoly if and only if $a/v \in \left(1 + \frac{\sqrt{6}}{3}, 1/(\sqrt{2} - 1)\right)$ and higher industry profit if and only if $a/v \geq 1$. But it always produces more social welfare.

**Proof:** By routine calculation, the net consumers’ surplus in the monopoly can be found as

$$\int_0^{\min(\frac{(a+v)/2a, 1})} [v + a(1-t)]dt - \max\{(a + v)/2, v\} \min\{(a + v)/2a, 1\} = \min\{(a + v)^2/8a, a/2\}.$$  

In comparison with (27), it can be seen that the monopoly yields more net consumer surplus if and only if $a/v \in \left(1/(\sqrt{2} - 1), 1 + \sqrt{6}/3\right)$. By (30) and (31), the monopoly profit is

$$(a + v)^2/4a \quad \text{if } a/v \geq 1 \quad \text{and} \quad v \quad \text{if } a/v \leq 1,$$

which is smaller than duopolistic industry profits (28) if and only if $a/v \geq 1$. The total surplus in the monopoly is

$$\int_0^{\frac{(a+v)/2a}{a/v}} [v + a(1-t)]dt = 3(a + v)^2/8a \quad \text{if } a/v \geq 1,$$

$$\int_0^1 [v + a(1-t)]dt = v + a/2 \quad \text{if } a/v \leq 1.$$

They are always smaller than duopolistic total surplus in (29). ☺️
Comparison with cartelization

Since FOCs with respect to per-unit prices in cartelization are the same as that of two-part tariff competition as demonstrated in the previous section, the cartel also sets prices equal to marginal cost, i.e., \( p_x = p_y = c \). With the utility specification of (24), the zero point of (10) and its counterpart have the feature that \( T_{x1} = (a + v)/2a > T_{y1} = (a - v)/2a \), which implies that partially supplying the market is not the best interests of the cartel as we have discussed. Substituting the utility specification of (24) into (19), it shows that the cartel supplies the whole market and chooses the marginal consumer at \( T = \frac{1}{2} \). The cartel definitely wants to choose both entry fees equal to the surplus of the marginal consumer, i.e., \( e_x = e_y = v + a/2 \). Thus, in the light of (26), we reach another proposition.

**Proposition 11.** With utility specification (24), duopolistic competition has the same outcome as cartelization if \( a/v \in (2, +\infty) \) but it has lower entry fees than cartelization if \( a/v \in (0, 2) \).

Noting that when \( a/v \in (2, +\infty) \), the equilibrium marginal consumer is at the kink that \( \bar{T}_x = (e_y - v)/a = \frac{1}{2} = \bar{T}_y \), so we have an example of Proposition 7.

Comparison with uniform pricing

As pointed in the previous section, the nondegenerated symmetric equilibrium of uniform pricing must be \( p_{x2} = p_{y2} = p_2 \). Since (20) gives \( T_x = [a + v(p_x) - v(p_x)]/2a \), we have \( T_x = \frac{1}{2} \) in equilibrium. Recalling \( X(p_x, T_x) = x(p_x)T_x \) and (22), and setting the right-hand side of (23) equal to zero, it turns out that the equilibrium price \( p_{x2} \) is characterized by

\[
x(p_{x2}) + (p_{x2} - c)\frac{\partial x(p_{x2})}{\partial p_x} - (p_{x2} - c)x(p_{x2})^2/a = 0.
\]

**Proposition 12.** With indirect utility functions (24), two-part tariff competition results in lower per-unit prices but higher consumers’ surplus and firm profits and in turn more social welfare than uniform pricing competition.

**Proof:** In the equilibrium determined by (32), prices must be greater than marginal cost. At the same time, consumer with taste parameter \( t \) or \( 1 - t \) has surplus \( v(p_2) + a(1 - t) \) so that all consumers’ surplus is

\[
2 \int_0^{0.5} [v(p_2) + a(1 - t)]dt = v(p_2) - 3a/4.
\]

Noting \( v(p_2) < v \), it can be seen
that two-part tariff competition results in more net consumers’ surplus than uniform pricing competition through comparing the above equation with (27).

Under uniform pricing, each firm earns profit \((p_2 - c)x(p_2)/2\). Recalling \(\partial x(p_2)/\partial p_i < 0\), (32) indicates that when \(a/v \in (0, 2)\) the industry profit of two-part tariff competition = \(a > (p_2 - c)x(p_2)\) = industry profit under uniform pricing. When \(a/v \in [0, +\infty)\), the industry profit in two-part tariff competition = \(v + a/2 = \int_c^\infty x(p)dp + a/2 > \int_c^x x(p)dp > (p_2 - c)x(p_2)\).

Since both consumers’ surplus and firm profits are larger in two-part tariff competition, so does social welfare.

Monopolistic two-part tariffs definitely make the marginal consumer worse off in contrasted to uniform pricing since the monopolistic entry fee drives his surplus to zero (see Philips (1982), Wilson (1993), Yin (2000)). However, the marginal consumer is likely better off in a two-part tariff duopoly than in a uniform pricing duopoly. In this example, the marginal consumer’s net surplus under the two-part tariff regime is \(v - a/2\) if \(a/v \leq 2\) while the surplus under the uniform pricing regime is \(v(p_2) + a/2\). Thus, if \(v > a + v(p_2)\), the two-part tariff makes him better off.

3.2) Example 2

This example assumes that the indirect utility is in a multiplication form of

\[
v_x(p_x, t) = (1 - t)(b - p_x)^2/2 \quad \text{and} \quad v_y(p_y, t) = t(b - p_y)^2/2,
\]

where \(b\) is constant satisfying \(b > c_x\) and \(b > c_y\). With such a specification, a consumer’s demand for a product, if he chooses to purchase it, is

\[
x(p_x, t) = (1 - t)(b - p_x) \quad \text{and} \quad y(p_y, t) = t(b - p_y).
\]

Thus, the demand is linear but when the taste parameter increases the \(x\)-good demand curve shifts inwards and the \(y\)-good demand curve shifts outwards. For consumers in the \(x\)-good market (\(y\)-good market) with larger \(t\) are smaller buyers of good \(x\) (larger buyers of good \(y\)).

By using (34), we obtain

---

8 The discussion from here until equation (37) does not need the assumption that \(c_x = c_y\). So, I relax it now.
\[ X(p_x, T_x) = \int_0^{T_x} (1 - t)(b - p_x) \, dt = (b - p_x)(2 - T_x)T_x/2. \]  

(35)

Substituting (34) and (35) into (9), routine calculation provides

\[ p_x = c_x + (b - c_x)T_x/2, \]  

(36)

which implies above-marginal-cost pricing. The reason for this above-marginal-cost price has been discussed earlier; that is, the demand of marginal consumer, \((b - p_x)(1 - T_x)\), is smaller than the average demand, \((b - p_x)(2 - T_x)\).

To find equilibrium, I show first that there is no partial-cover equilibrium. As discussed in the previous section, partial-cover equilibrium can only realize at \(T_x = T_{x1}\).

Substituting (33) and (34) into (10) and setting it to zero to obtain

\[ T_{x1} = (b + p_x - 2c_x)/2(b - c_x). \]  

(37)

Since \(p_x > c_x\), we have \(T_{x1} > 1/2\). Through the same process, we can also show that \(T_{y1} < 1/2\). As equilibrium requires \(T_x \leq T_y\), it is clear that \(T_x = T_{x1}\), \(T_y = T_{y1}\) cannot be equilibrium. In other words, each consumer is supplied by either firm.

To focus on symmetric full-cover equilibrium, I now add the assumption that \(c_x = c_y = c\) so that \(T_x = T_y = 1/2\) in equilibrium. Substituting them into (36) and its counterpart for \(p_y\) yields equilibrium per-unit prices

\[ p_x = p_y = c + (b - c)/4. \]  

(38)

To find equilibrium entry fees, we need to know whether equilibrium \(T_x\) is at \(T_{x2}\) or at \(T_{x2}\). By the utility specification of this example, it is easy to calculate that

\[ T_{x2} = [(b - p_x)^2/2 + e_y + (p_x - c)(b - p_x)][(b - p_x)^2 + (b - p_y)^2 + (p_x - c)(b - p_x)]/[(b - p_x)^2 + (b - p_y)^2 + (p_x - c)(b - p_x)]. \]  

(39)

If equilibrium is at \(T_x = T_{x2}\), then substituting \(T_{x2} = 1/2\) and the values of \(p_x\) and \(p_y\) in (38) into the above equation, it turns out that \(e_y = 3(b - c)^2/16\). However, for the marginal consumer \(T_y = 1/2\), his surplus from the consumption of good \(y\) is equal to \(9(b - c)^2/64 < 3(b - c)^2/16\). In other words, he is unwilling to pay an entry fee, \(e_y < 3(b - c)^2/16\). Thus, we reach a contraction, which implies that equilibrium cannot be \(T_x = T_{x2}\). Instead, the equilibrium must be \(T_x = T_{x2}\) and \(T_y = T_{x2}\). Substituting \(T_x = 1/2\) and the values of \(p_x\) and \(p_y\) in (38) into (39), we can obtain equilibrium entry fees

\[ e_x = e_y = 9(b - c)^2/64. \]  

(40)
Noting the marginal consumer’s indirect utility from consumption is also equal to \(9(b - c)^2/64\), he has zero net surplus after paying entry fee.

**Proposition 13.** With indirect utility functions (33), the full-cover symmetric equilibrium of two-part tariff competition results in above-marginal-cost per-unit prices (38) and entry fee (40), which drives the marginal consumer’s net surplus to zero.

To the net surplus of all consumers, it can be calculated as

\[
2 \int_0^{0.5} \left[ (1-t)(b - p_x)^2 / 2 \right] dt - 9(b-c)^2/64 = 9(b-c)^2/128.
\]

By (4), (35) and (38), the industry profit is

\[
2e_xF(\frac{1}{2}) + 2(p_x - c)X(p_x, \frac{1}{2}) = 9(b-c)^2/64 + \left[ c + (b-c)/4 - c \right][b - c - (b-c)/4](2 - \frac{1}{2})/2
\]

\[
= 9(b-c)^2/32.
\]

The total surplus is

\[
9(b-c)^2/128 + 9(b-c)^2/32 = 45(b-c)^2/128.
\]

**Comparison with monopoly**

From (37), it is easy to see that if the market has only one firm, it determines the marginal consumer at \(T_x = \min\{ (b + p_x - 2c)/2(b - c), 1 \} \). Using (36), the monopoly equilibrium can be characterized as

\[
T_x = 2/3, \quad p_x = c + (b - c)/3, \quad e_x = 2(b-c)^2/27.
\]

From these results and equations (38) and (40), we have the proposition below.

**Proposition 14.** If indirect utility functions are those of (33), duopolists supply more consumers with lower per-unit prices but higher entry fees than a monopolist, resulting in more net consumers’ surplus, industry profit and social welfare.

**Proof:** Recalling (38), (40) and (44) and the duopolists supply all consumers, the conclusions on market size, per-unit price and entry fee are immediate. Using (44), the net consumers’ surplus in the monopoly can be calculated as

\[
\int_{0}^{(2/3)} \left[ (1-t)(b - p_x)^2 / 2 \right] dt - (2/3)[2(b-c)^2/27] = 4(b-c)^2/81.
\]

Using (4) and (44), the monopoly profit is \(e_xF(2/3) + (p_x - c)X(p_x, 2/3)\)

\[
= (2/3)[2(b-c)^2/27] + \left[ c + (b-c)/3 - c \right][b - c - (b-c)/3](2 - 2/3)/3 = 4(b-c)^2/27.
\]
Comparing them with (41)-(42) welfare conclusions are obvious.

Again, the reason for a lower entry fee in the monopoly is because the monopolist serves more than a half of all consumers.

**Comparison with cartelization**

As discussed in the model, the cartel follows the same per-unit pricing rule as (36). Since we have demonstrate that $T_x > T_y$ in this utility specification, the cartel would choose to supply the whole market with $T = \frac{1}{2}$, and charge the entry fees equal to the surplus of marginal consumer, i.e., $e_x = e_y = 9(b - c)^2/64$.

**Proposition 15.** With indirect utility functions (33), duopolistic competition equilibrium is exactly the same as the cartelization outcome.

Noting the duopolistic competition equilibrium is at $\bar{T}_x = \bar{T}_y = \frac{1}{2}$, it reconfirms Proposition 7.

**Comparison with uniform pricing**

With indirect utility specification (33), the marginal consumer under uniform pricing is $T_x = (b - p_x)^2/[(b - p_x)^2 + (b - p_y)^2]$ by (20), which implies that $T_x = \frac{1}{2}$ in symmetric equilibrium. Applying (33) and (34) to (22), it gives $dT_x/dp_x = -(1 - T_x)(b - p_x)/[(b - p_x)^2 + (b - p_y)^2]$. Substituting this and (34)-(35) into (23), and setting it equal to zero, we have an equation characterizing the symmetric equilibrium $p_{x2} = p_{y2} = p_2$ that

$$(b - p_{x2})(2 - T_x)T_x/2 - (p_{x2} - c_x)(2 - T_x)T_x/2 + (1 - T_x)^2(b - p_{x2})^2/[(b - p_{x2})^2 + (b - p_{y2})^2] = 0.$$  

Since $T_x = \frac{1}{2}$, it leads to symmetric solution $p_{x2} = p_{y2} = p_2 = (3b + 4c)/7$.

Using these equilibrium prices, routine calculation gives

Consumers’ surplus = $\int_0^{0.5} (1 - t)(b - p_x)^2 dt = 3(b - p_x)^2/8 = 9(b - c)^2/49$.

Industry profit = $2(p_2 - c)X(p_2, \frac{1}{2})/2 = 6(b - c)^2/49$.

Total social welfare = $15(b - c)^2/49$.

In the light of (38), (41)-(43), we have
Proposition 16. With indirect utility functions (33), the two-part tariff competition equilibrium leads to lower per-unit prices and net consumers’ surplus, higher industry profits and social welfare than the uniform pricing competition.

4. Discussion of the Two Examples and Concluding Remarks

Since two-part tariff competition in both examples results in symmetric full-cover equilibrium, whether it maximizes social welfare depends on whether the equilibrium per-unit prices are equal to marginal cost. While the first example shows marginal-cost pricing and social welfare maximum, the second does not. Like the case of two-part tariff monopoly, social welfare maximization does not implies consumers are better off. As illustrated in Example 1, welfare-maximizing two-part tariff competition can coincide with the achievement of two-part cartelization.

Related to this feature is that two-part tariffs may reduce the severeness of competition, evidenced by the coincidence of the outcomes of duopolistic competition and cartelization in Example 1 when \( a/v \geq 2 \) and in Example 2. Noncorporative firms in the process of seeking their own interests can end up with corporative outcomes under a two-part tariff regime. For symmetric partial-cover equilibrium, the coincidence is unconditional in our model. But this seems not too surprising since firms do not compete head to head in such equilibrium.\(^9\) More interesting is symmetric full-cover equilibrium, where firms appear to enter the rival’s market range and steel their business. In such imperfectly competitive equilibrium, the prices are unconditionally equal to the prices in cartelization. The only possible difference is entry fees, where imperfect competition may force firms to cut off entry fees. But such a price war does not happen when the imperfectly competitive equilibrium is at the kink of profit functions. What Examples 1 and 2 demonstrate is that there is a wide range of conditions to make the imperfectly competitive equilibrium realize at the kink. This raises an issue that whether the practice of two-part tariffs by itself is collusive to certain extend or whether two-part tariffs facilitate collusion in oligopoly.

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\(^9\) It still raises a question that whether firms can avoid direct competition through two-part tariffs.
Conventional belief in monopoly is that it produces the largest industry profits in contrast to other industry structures even without economies of scale. However, this is no long true in our examples. In Example 1, the monopoly has larger industry profits than the duopoly if and only $a/v \leq 1$ (product differentiation or consumer heterogeneity is small) while in Example 2 monopoly profits are definitely smaller. This phenomenon is due to a larger firm-level market size in the monopoly than in the duopoly, which makes a monopolistic firm have to charge a lower entry fee than a duopolistic firm. As in Example 1, the per-unit prices are the same in both market structures so that the marginal consumer’s surplus (before entry fee) in the monopoly is lower, which forces the monopolist to levy a lower entry fee. In Example 2, the monopoly price is marginally higher, but the significant firm-level market size difference still results in lower marginal consumer’s surplus in the monopoly than in the duopoly. On the other hand, this lower entry fee may lead to more net consumers’ surplus in the monopoly than in the duopoly under certain conditions. The condition is $a/v > 1$ in Example 1 but in Example 2 it is unconditional. Although all consumers as a whole may be better off by paying a lower entry fee, some consumers close to one terminator of the consumer distribution are worse off. A direct implication of this observation is that the entry of new firms may not benefit consumers, which calls on to exam the doctrine that “more competition is better to consumers” on a case-by-case base in a world of two-part tariffs.

More contradicting effects of two-part tariff competition on the net consumers’ surplus can be seen through comparing it with uniform pricing competition. In Example 1, the two-part tariff does produce more net consumers’ surplus than uniform pricing but it is not the case in Example 2. Although two-part tariffs induce lower per-unit prices than uniform pricing in both examples, the extra entry fees create ambiguity of net effects on consumers’ surplus. However, two-part tariffs definitely help firms earn more profits than uniform pricing in both examples. Corts (1998) finds that third-degree price discrimination may lead to lower profits than uniform pricing so that “imperfectly competitive firms may wish to avoid price discrimination and the all-out competition that may ensure, perhaps through commitments to uniform pricing”. This seems not the case for two-part tariffs.
Since firms under two-part tariffs are definitely better off than uniform pricing, they certainly like to commit to two-part tariffs, if possible, to enhance the profit performance.

The model presented in this paper can be easily extended to more general oligopolistic industry structure. For instance, if there are \( n \) firms in an industry, we can consider they distribute on a Salop (1979) circle rather than locate at the ends of a straight line. Then, except for that each firm must directly compete with its neighbors on each side, other elements of modeling are similar. However, such a framework would be more suitable for further extension to include firm location choice under a two-part tariff regime.

**APPENDIX**

*Equilibrium of Example 1*

Summarize reaction curve as follows.

\[
\begin{align*}
\text{Case (i), } a/v &\geq 1, \quad e_x = \begin{cases} 
(a + e_y)/2 & \text{if } e_y \leq (a + 4v)/3 \\
2v - e_y & \text{if } (a + 4v)/3 \leq e_y \leq (a + 3v)/2 \\
(a + v)/2 & \text{if } e_y \geq (a + 3v)/2 
\end{cases} \\
\text{Case (ii), } \frac{1}{2} \leq a/v \leq 1, \quad e_x = \begin{cases} 
a + 2v - e_y & \text{if } (a + 4v)/3 \leq e_y \leq a + v \\
2v & \text{if } e_y \geq a + v 
\end{cases} \\
\text{Case (iii), } a/v \leq \frac{1}{2}, \quad e_x = \begin{cases} 
\max\{(a + e_y)/2, e_y - a\} & e_y \leq a + v \\
v & e_y \geq a + v
\end{cases}
\]

Similar formulas are applicable to firm \( y \)'s reaction function. Consider Case (i) now.

(a) If the reaction curves of the two firms intersect on the segments \( e_x = (a + e_y)/2 \) and \( e_y = (a + e_x)/2 \), they yield \( e_x = e_y = a \). Substituting them into the constraints of these segments, \( e_x \leq (a + 4v)/3 \) and \( e_y \leq (a + 4v)/3 \), leads to the condition for equilibrium that \( a/v \leq 2 \).

(b) If the intersection is on the segments \( e_x = (a + e_y)/2 \) and \( e_y = a + 2v - e_x \), then \( e_x = 2(a + v)/3 \) and \( e_y = (a + 4v)/3 \). Substituting the solution into the constraint on \( e_x \), \( (a + 4v)/3 \leq e_x \leq (a + 3v)/2 \), leads to the condition for equilibrium that \( 3 \leq a/v \leq 5 \).

(c) If the intersection is on the segments \( e_x = (a + e_y)/2 \) and \( e_y = (a + v)/2 \), then \( e_x = (3a + v)/4 \) and \( e_y = (a + v)/2 \). Substituting them into the constraints of these segments, \( e_x \geq (a + 3v)/2 \) and \( e_y \leq (a + 4v)/3 \), leads to the condition for equilibrium that \( a/v = 5 \).
(d) If the intersection is on the segments \( e_x = a + 2v - e_y \) and \( e_y = a + 2v - e_y \), then any \( e_x \) and \( e_y \) satisfying \( e_x + e_y = a + 2v \) can be equilibrium. But there are constraints for these two segments that \( (a + 4v)/3 \leq e_x \leq (a + 3v)/2 \) and \( (a + 4v)/3 \leq e_y \leq (a + 3v)/2 \), which imply \( a/v \geq 2 \).

(e) If the intersection is on the segments \( e_x = a - 2v - e_y \) and \( e_y = (a + v)/2 \), then \( e_x = (a - 5v)/2 \), which contradicts the constraint on \( e_x \) that \( e_x \geq (a + 3v)/2 \). Thus, \( e_x = (a - 5v)/2, e_y = (a + v)/2 \) cannot be equilibrium.

(f) By the argument similar to (e), \( e_x = e_y = (a + v)/2 \) cannot be equilibrium.

Note that the equilibria \( e_x = 2(a + v)/3, e_y = (a + 4v)/3 \) in (b), \( e_x = (3a + v)/4, e_y = (a + v)/2 \) in (c) are on the line \( e_x + e_y = a + 2v \). Applying the above process to Cases (ii) and (iii), we can find that when \( a/v \leq 2 \), there is a unique equilibrium that \( T_x = T_y = \frac{1}{2} \) and \( e_x = e_y = a \). When \( a/v > 2 \) there are a continuum of equilibria that \( e_x + e_y = a + 2v \) as far as \( e_x \) and \( e_y \) satisfy \( (a + 4v)/3 \leq e_x \leq (a + 3v)/2 \) and \( (a + 4v)/3 \leq e_y \leq (a + 3v)/2 \). The marginal consumer in equilibrium can be determined by \( T_x = T_y = (e_y - v)/a \).
REFERENCES


Figure 1 (a) $a/v \leq 2$

Figure 1 (b) $a/v > 2$